PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 69 DECEMBER, 1943 NO. 10

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

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Printed in the United States of America

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

GEOLOGY IN HIGHWAY ENGINEERING

BY MARSHALL T. HUNTTING,[1] ESQ.

SYNOPSIS

Within recent years engineers and geologists have worked to bring about a closer bond between their respective professions. Engineering geology studies have been conducted in many parts of the world by goverment agencies, engineering societies, and independent workers.

Geology has entered into highway engineering problems in many ways, and study of geological features has helped to solve many highway construction and maintenance problems. However, many more such problems could be solved profitably by the use of geologists' services. Twelve of these problems will be treated in this paper, and examples will be given to show how some of them are being solved through the cooperative work of geologists and highway engineers. It is the purpose of this paper to help both engineers and geologists to recognize more frequently those situations in which geology does enter into highway engineering.

INTRODUCTION

In contrast to the rapid advance of engineering sciences in the fields of statics, dynamics, and hydraulics within the past century, there was a general lag in advancement of those phases of engineering involving earth materials and earth processes, which together constitute the basis of the science of geology. Comparatively recently, however, has come a fuller appreciation of the value of scientific study of the earth materials upon which must rest the foundations of the structures planned and constructed by engineers. As a result many engineers have worked to perfect a better bond between geology and engineering.

Engineering geology studies, many of which are applicable to highway engineering problems, have been initiated in Europe and in America by governmental and municipal agencies, by engineering societies, and by individual workers. Reports of the influence of geology on roads appear in English publications as early as 1888. Since that time, especially within the past few

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May 1, 1944.**

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decades, the results of geological investigations into specific highway engineering problems have been published both in Europe and America, but apparently no treatment of the subject as a whole is available.

The work of the geological surveys of several of the states in the United States will be used to illustrate some geological applications to highway engineering.

Problems

Of the multitude of problems confronting highway engineers many involve geologic factors and thus should command the attention of geologists. No attempt will be made herein to present a complete study of any of the problems; instead, the purpose is to indicate some of those instances in which geologists' services might be profitably engaged. These follow:

(1) Locating suitable road-surfacing material pits and quarries;
(2) Determining the suitability of various earth materials for surfacing, concrete construction, and other highway uses;
(3) Subgrade treatment and classification;
(4) Frost-heave problems;
(5) Predicting the character of material to be excavated;
(6) Preventing and correcting landslides;
(7) Appraising the competency of materials for bridge foundations;
(8) Judging the desirability of bridge sites and road sites with respect to possible changes of stream channels;
(9) Investigating proposed tunnel sites;
(10) Investigating and predicting subsurface water conditions and locating underground water supplies;
(11) Evaluating mineral lands to be bought or condemned for highway location; and
(12) Acting as witness in litigation.

Although highway activities have always claimed some attention from geologists, many, if not most, of the foregoing problems have been insufficiently considered. Economic and other factors will determine the importance of many of them, and some will be relatively more important than others, depending upon several factors, of which the geologic character of the terrain traversed by a given road is probably the most significant. In some instances geological investigations have been made by independent consulting geologists called in for specific assignments, and some have been made as a part of the routine duties of men who were an integral part of the state highway departments, but more frequently they have been made by the state geological surveys in response to highway department requests for consultory advice on special problems.

Location of Suitable Road-Surfacing Material Pits and Quarries

Probably the most obvious and certainly the most widely recognized application of geology to highway engineering is in making materials surveys, which involve locating, and determining the quality, quantity, and the natural fea-

tures which affect the exploitation of gravel, stone, clay, and other types of material for subgrade construction, surfacing, and concrete construction. Of the

> "* * * total annual expenditure of $500,000,000 [for roads in the United States], probably over half is spent for materials and their transportation to the construction projects, and it is in the cost of these materials that the largest savings can be effected" (1).[2]

The possibility of effecting financial savings and of obtaining better road materials through materials surveys is almost universally recognized, as are also the advantages of having the surveys made by men who have specialized in geology and have made the study of earth materials their profession. Obviously earth materials surveys can best be made by specialists, who, after learning from the engineers specifically which rock types have the physical and chemical specifications best suited to given needs, can then apply their understanding of the geological phenomena controlling the origin, subsequent alteration, and present physiographic expression of those rock types to locate, classify, and delimit commercial deposits for highway use. The geologist, through his knowledge of the glacial, stratigraphic, petrologic, and physiographic history of a given area, should be able to locate, easily, bodies of any type of rock present, without resorting to the too often used method of haphazard test pitting, which is an expensive and tedious process at best.

In some sections of the United States crushed rock is used to a large extent for surfacing and other construction and maintenance work. Selection of sites in rock to be quarried and crushed has been guided in some instances only by the character of plant site available, distance to place the crushed rock, and other such factors, without sufficient regard for the geologic factors involved. The extent, continuity, and uniformity of rock, where not exposed, are features predictable only on a geologic basis. The degree and extent of jointing and of weathering or other alteration are other such features.

Similarly, the nature of a deposit of sand, gravel, or other nonconsolidated material is determined by its geologic origin and so is predictable only when that origin is known. Geologists have been most widely employed in highway work for locating and testing such deposits.

Among the state organizations conducting materials surveys is the Louisiana Geological Survey, which locates and maps deposits of sand, gravel, rock, and shell, and which, in 1941, published a bulletin (2) on sand and gravel deposits of Louisiana. This work was financed partly by the State Highway Department and has since been put to considerable use not only by that department but also by various military groups in the state for building roads in the vicinity of the army cantonments.

In Texas the Bureau of Economic Geology of the University of Texas, in Austin, has cooperated with the State Highway Department in the location of road materials. On most of their projects geophysical prospecting apparatus of the resistivity type has been used to determine the thickness of sand and gravel beds.

The Areal and Engineering Geology Division of the Illinois Geological

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

Survey has conducted comprehensive studies in cooperation with, and for the benefit of, the Highway Division. One such study was of the limestone resources of the state, the results of which (3) were published in 1925. Another was a study of sources of material, particularly gravel and thin limestone deposits, for use on local roads. Maps of clay deposits in part of southwestern Illinois were made for use in obtaining clay materials for stabilized roads.

Wisconsin is a pioneer and leader in materials survey programs. As early as 1907 its Geological Survey was appropriated $10,000 by the state legislature for investigations and experimental work in road building. This work has been continued, and the survey now assists the highway engineers in every way possible, including the following, quoted from a letter by E. F. Bean:

(*a*) Road materials surveys. For nearly all construction projects, surveys are made for surfacing materials, sand-gravel fill, and clay binder for gravel.
(*b*) Classification of excavation. In some cases division engineers are consulted on classification of material to be excavated.
(*c*) Subgrade treatment. Conferences and consultations in the field have been held.
(*d*) Litigation. The state geologist has been a witness for the state on cases involving excavation, gravel, and limestone.

The road materials surveys have been especially fruitful, and for this work the Wisconsin Geological Survey is given an appropriation of $15,000 per yr.

Wisconsin Materials Survey Procedure.—The Wisconsin Highway Department has mobile crushing, screening, and washing plants which are moved to sites located by Geological Survey men at the request of the county highway departments. The highway department gives to the state geologist the exact specifications of the rock required, the quantity needed, the road on which it is to be used, and the use to which it is to be put. A geologist immediately goes into the field to investigate road materials in the near vicinity of the work to be done. He locates a number of pit sites in material which will meet the highway specifications. He then examines in detail each site selected to determine tonnage available, amount of overburden, cost of quarrying, and the pertinent geological features. Of each site he makes maps on which his findings are illustrated. Out of a probable eight or ten sites to be selected, the geologist may recommend two or three which will then be more extensively explored by an engineer to determine which deposit shall be developed.

Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses

Physical tests to determine the suitability of rocks for road surfacing are many and varied, and in the main they are sufficient in themselves. It is in determining the reason why certain rocks are durable and others are not that the geologist can be of service. This is not a question of academic interest only; on the contrary, it is one of considerable practical value, in that through recognition of the critical factors the geologist, by petrographic examination, can indicate quickly whether or not a given rock is likely to be satisfactory for a specific use.

High toughness is usually associated with traprock, and, in general, the fine grained volcanic rocks are harder and tougher than the coarser grained plutonic rocks. The sedimentary rocks vary widely in physical characteristics, but they are generally softer and less tough than the igneous rocks. Limestone and dolomite have greater toughness and are equal in hardness to the average sandstone, as determined by the standard tests. Of the metamorphic rocks, marble is generally neither as hard nor as tough as its unmetamorphosed equivalent, limestone. In most instances the foliated metamorphic rocks are comparatively soft and of low toughness, and as a rule they are rendered unsuitable for most road building purposes by their characteristic tendency to crush into long, thin, or flat fragments.

The hardness and toughness of these rocks are dependent upon the following fundamental properties: (*a*) Mineral composition (relative abundance of various minerals) and chemical composition (hardness, toughness, and cleavage of individual minerals); (*b*) texture (size and shape of grains and porosity); (*c*) freshness of minerals (degree of weathering or hydrothermal alteration of original minerals); and (*d*) jointing and fracturing. These fundamental properties are easily observed by the trained eye alone or with the aid of a good hand lens or petrographic microscope.

Failure of concrete by disintegration presents problems in which the importance of investigating the properties of the aggregate rocks has been minimized far too greatly. Attempts to prevent concrete failures were originally concentrated on varying the water content of the mix, experimenting with different types of cement and various cement-aggregate proportions, and adding special ingredients to the mix. Little attention was paid to the characteristics of the aggregate rocks. Since 85% to 90% of all concrete consists of aggregate, the quality of the concrete undoubtedly is governed to a marked degree by the quality of the aggregate, and the recognition of this fact has stimulated some fruitful research along this line. In 1931 Oliver Bowles reported (4) that failure of some concretes had been traced to physical defects in the rock aggregate, and in this connection he stated,

> "Physical tests of abrasion resistance and crushing strength should be supplemented by microscopic examination to determine soundness and the presence or absence of undesirable minerals. Microscopic examination is particularly valuable in recognizing partial alteration of the original minerals, which is accompanied by the formation of secondary minerals, usually of inferior quality."

More specific are the warnings of J. C. Pearson and G. F. Loughlin in 1923 (5) reporting the failure, in less than a year and a half, of concrete using the plagioclase feldspar labradorite for aggregate. Microscopic examination showed that the feldspar was largely altered to the minerals, laumontite, kaolin, and calcite, in decreasing order of abundance; and it was indicated that these three minerals were responsible in the same order for the disintegration of the concrete. Feldspar aggregate which did not cause failure was found to be unaltered. Strength tests applied to test specimens failed to indicate the unstable nature of the altered feldspar aggregate, thus emphasizing the advisa-

bility of having petrographic analyses made of questionable aggregate material previous to its use.

Further recognition of the rôle of inferior aggregates in concrete failures is noted in a highly informative paper (6) by Thomas E. Stanton, M. Am. Soc. C. E., Materials and Research Engineer for the California Division of Highways. Extensive tests conducted by him demonstrated that some concrete failures result from excessive expansion of concrete caused by chemical reaction between high alkali cements and certain mineral constituents in some aggregates, such as some shales, cherts, and impure limestones. Opaline silica, apparently the trouble-causing mineral, was identified in aggregates of the defective concretes. The mineral identifications were made with the microscope by a geologist, Allen H. Nicol, Mineral Technologist for the Aggregate and Soils Department of the California Division of Highways.

In 1941 the Washington State Department of Highways engaged H. A. Coombs as consulting geologist to determine the cause of failure of highway concrete at several places in the state. Late in the year Mr. Coombs reported (7) the results of his investigations into the cause of disintegration of the concrete in several bridges near Ohanapecosh on the White Pass Highway. Test procedures patterned after those developed by Mr. Stanton in California were used to prove the failures to be the result of excessive concrete expansion. The expansion was thought to result from chemical reaction between cement and aggregate. Thin section study with the petrographic microscope combined with fluorescent tests with ultraviolet light proved the reactive aggregate material to be pebbles of andesite. Further work should indicate what percentage of the deleterious andesite can be used in aggregate without causing undue expansion, and all rock deposits exceeding that percentage of andesite should be condemned as sources of aggregate for concrete.

Subgrade Treatment and Classification

Since the weight of traffic is supported not on the pavement but rather on the subgrade, it is obvious that the properties of the subgrade materials will exert a tremendous influence on the success or failure of the surface. The most common subgrade material is "soil," a term which, depending upon its definition, may mean only that part of the mantle which supports plant roots, or it may be used to cover a wide variety of unconsolidated earth materials ranging from gravel through sand, silt, and clay, and including organic deposits. The term will be used in its broadest sense to include all the unconsolidated mantle overlying solid bedrock.

It is well known that soils possessing obviously different physical properties give equally different performances as road building materials. Recognizing this the Public Roads Administration (PRA) and most state highway departments have set up soil testing divisions whose work has been to classify soils, to test their physical properties, to correlate test determined physical properties with performance records, and to make recommendations on the basis of their findings. Soil studies conducted by them and other interested organizations since the 1920's have brought forth a wealth of information which has been

successfully applied to the solution of subgrade problems. Most valuable, perhaps, have been their soil classifications based on physical properties and correlated with performance records.

A general soil classification such as that developed by the PRA (8) is satisfactory for most applications of soil mechanics to highway engineering as pointed out by K. B. Woods, Assoc. M. Am. Soc. C. E., and R. R. Litehiser, M. Am. Soc. C. E. (9), but in exceptional cases there is found need for a more specific classification such as one based on mineralogical composition of soil. Some of the effects of mineral composition on the behavior of clays and soils were reported in 1940 by G. E. Ekblaw and R. E. Grim (10). They classify the clay minerals under the three groups—(*a*) kaolinite group, (*b*) montmorillonite group, and (*c*) illite (11) group of sericite-like minerals. Each group has well-defined but vastly different capacities to absorb water, possess exchangeable bases, and break down into extremely small particles which will determine to a large degree the plasticity and permeability of soils containing these clay minerals. Plasticity and permeability in turn are extremely influential in determining compaction, soil slippage, and frost heave—features of considerable importance to the highway engineer.

In other instances where a soil classification based on physical properties is not entirely adequate, a supplementary classification may be useful, based on the soil's origin such as the following:

I. Residual soils.
II. Colluvial soils.
III. Organic soils.
IV. Transported soils—*A*, alluvial; *B*, lacustrine; *C*, marine; *D*, glacial; and *E*, aeolian.

The soils falling under a given heading in this classification will not have all their physical properties alike, but it will be found that some of their properties will be remarkably uniform over wide areas.

Nothing can replace the engineers' physical tests to identify soil types and to determine their suitability in subgrades on the basis of performance, but testing and sampling will be most effective and economical only when something is known of the soil strata's probable uniformity and regularity—features which are dependent in large part upon the origin of the soil, and the closely related mineralogical and chemical composition of the soil, soil particle size and shape, local topography, subsurface water conditions, and climatic conditions. The geologist's understanding of these features will help in interpreting sample test results and will enable him to suggest where samples should be taken. Thus, the direct economy in the use of geologists is in reducing the number of actual tests.

Geologists have played a leading rôle in the soil testing divisions of many state highway departments and have lent substantial assistance in others.

Outstanding is the work of the Illinois State Geological Survey's Areal and Engineering Geology Division, established in1927 and headed by Mr. Ekblaw. According to R. F. Legget, Assoc. M. Am. Soc. C. E. (12), this organization is

unique in being the only state or national geological organization to recognize, to such an extent, the interdepending relations of geology and engineering. In a letter, Mr. Ekblaw states that his division has given advice on scores of problems involving landslides, rockfalls, slump, sod creep, and similar movements of earth above, beside, or beneath highways; heave and collapse of various kinds of pavements; "mud pumpings"; stability of fills, both as regards material in and foundation below the fill; treatment of peat bogs; foundation material for piers and abutments for bridges, overpasses, and underpasses; quality and location of aggregate material for concrete or other types of pavements; all kinds of drainage conditions; and location of highway routes to avoid undesirable conditions. Some of the advice has been preventive, given before construction was undertaken, and some has been curative, given after damage occurred.

Formerly the Michigan Geological Survey Division had direct cooperative relations with the State Highway Department. It conducted surveys for road gravel and road material and made test borings for swamp and marsh fills. The latter work in 1925 was transferred to the newly created Soil Engineering Branch of the State Highway Department's Testing Division. The soil classification used by them is that of the international system, described in the 1938 *Yearbook of Agriculture*, Soils and Men.

In Ohio two geologists are employed in the soil sampling and testing work of the Soil Section of the State Highway Testing Laboratory. The data and recommendations resulting from the soil, foundation, and geologic studies made on all projects involving federal aid funds and on all major state projects are thought to result in savings to the state amounting to several times their cost. Detailed description (13) of the work of their geologists and soil engineers may be found in the *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, which also includes articles describing the aims and methods of soil investigations in several other states.

H. A. Buehler, State Geologist for Missouri, believes that his state's highway department probably utilizes geology to a greater extent than any other highway department in the nation. The state geologist has been an ex-officio member of the State Highway Commission since its inception. The department's testing division organized a geological investigation branch, which has been the center around which soil stabilization has grown. Using one geologist for every two field divisions, the headquarters office has five geologists, all of whom are field experts in stabilization work.

Frost-Heave Problems

Frost heave might properly be discussed under the previous heading ("Subgrade Treatment and Classification"), but it is thought to be a problem of sufficient weight to merit separate treatment. Frost heave is a phenomenon to be dealt with in highway construction and maintenance over a large part of the United States, and with improved facilities for snow removal annually increasing the mileage of road left uncovered and open to frost attack, the need for frost-heave studies becomes more and more apparent.

Obviously the features of climate, rainfall, and temperature will broadly control the limits of the area in which frost action is of consequence. Recognizing this, J. A. Sourwine, Senior Highway Engineer, PRA, in 1930 concluded from a study (14) of climatological records that the area subject to frost damage to highways includes Maine, New Hampshire, Massachusetts, Rhode Island, Connecticut, New Jersey, New York, Pennsylvania, Ohio, West Virginia, Kentucky, Indiana, Michigan, Wisconsin, Iowa, Illinois, Missouri, northern Oklahoma, northern New Mexico, northern Arizona, Utah, Colorado, Kansas, Nebraska, South Dakota, North Dakota, Wyoming, Montana, Idaho, eastern Oregon, and eastern Washington. Within this area frost heave will be a problem only in those places where drainage conditions and soil characters are so combined as to make excessive differential frost action possible.

Laboratory and field studies in connection with frost heaving have been active for many years and much information on the subject has accumulated. The first explanation of the true nature of frost heaving appeared in 1918 when Stephen Taber (15), State Geologist and Professor of Geology at the University of South Carolina, at Columbia, showed that heave was the result not of expansion of water on freezing but of growth of lenses or layers of ice crystals at the frost line. He found that in order for ice to accumulate in layers a supply of water must be maintained to the zone of freezing either by a water table so high that it is above the frost line, or by capillary rise from a source below. The first source of supply can be removed comparatively easily by drainage, but the second is not so easily disposed of.

It has been shown that in many, if not most, instances of frost-heave disturbances clay is a major constituent of the subgrade material. The clay acts as the medium through which water rises from the water table by capillarity until it reaches the frost line, where it freezes and causes an irregular upheaval of the road surface. The subgrade soil profile ordinarily varies widely within small areal limits; consequently the amount of water available for freezing under the road surface will vary in a short distance, thus causing differential uplift and a rough road.

A comparatively recent suggestion is that mineral content is an important factor influencing the physical properties of clays subject to frost heave. It is known that two clays may have the same ultimate chemical composition and the same grade size distribution and yet possess quite different physical properties if their mineral compositions differ. Obviously then, a knowledge of the effects on the physical properties of a clay attributable to each constituent mineral is necessary in order to understand exactly why a given soil acts as it does. Such knowledge can then be applied to the planning of effective preventive measures. So far as the writer is aware, no research to determine the relation of clay mineral composition to frost heaving has yet been completed, but in this connection Mr. Grim has made some preliminary studies (16).

Suggested preventive methods against frost-heave damage have been many and varied. During the period from 1928 to 1933 the PRA, cooperating with the highway departments of Minnesota, Wisconsin, and Michigan, conducted a survey to correlate the various methods of different engineers. In the result-

ing report (17) Henry Aaron gives three classes of preventive measures. Additional preventive measures have been suggested in reports by H. F. Winn (18) and A. Scheidig (19). Selection of the best preventive measure in any given case must depend on economic considerations and on the soil profile in the area being considered; thus it must depend directly or indirectly on information furnished by soil surveys.

Prediction of Character of Material to be Excavated

Probably more disputes have arisen in civil engineering contracts through inadequate classification of material to be excavated than through any other feature. The cost of digging solid rock may be ten times or more the cost of digging an equal amount of earth (Fig. 1); consequently, a contractor should have the most accurate available estimate as to the volumes of each he will have to move, and the classification should be made in terms of the methods to be used to excavate the material. On large jobs estimates made by pure guesswork or by rule-of-thumb methods are certainly inadequate. Yardage estimates of solid rock and of unconsolidated material will be most reliable where the estimator, knowing the geologic processes operative in the accumulation of those materials, makes his estimates on the basis of that knowledge.

Fig. 1.—The Cost of Digging Rock May Be Ten Times That of Digging an Equal Volume of Earth

In excavating intrusive igneous rocks, such as granite, it may be assumed that the rock will remain quite uniform both at great depths and for long distances laterally with respect to properties affecting excavation, except for jointing and weathering. Jointing is generally best developed along the sides and top of the intrusive rock mass, near its contact with the intruded rock. However, a granite mass which has been deeply eroded and covered by sedimentary rocks probably would not be more jointed near its contact with the surrounding rock than elsewhere, thus illustrating the need for knowing the geologic history of a rock to be excavated. The degree and depth of weathering of granite, as well as of every other rock, depend upon the rate of weathering

and rate of erosion which carry away the weathered rock. Glaciated areas generally show little weathering in their rocks, owing to the comparatively recent vigorous scouring action of the ice.

Extrusive igneous rocks, such as basalt, may be extremely varied in nature over short distances laterally or vertically, or they may be quite uniform, depending upon their chemical composition, their mode of origin, the type of topography over which they flowed, their rate of cooling, and the amount of deformation, erosion, and weathering they have suffered. In basalt, jointing may be almost entirely lacking or may be very well developed, depending on the foregoing determinants. The degree of jointing very largely controls the cost of excavating such rock, and can be predicted fairly safely.

Unconsolidated earth materials (soils) may vary considerably according to their origin, but they are nearly all easily excavated. The important factor in considering such deposits is their depth to underlying solid rock. Within limits, the depth and shape of such deposits are reliably predictable by competent geologists.

The various sedimentary rocks and unconsolidated sedimentary deposits vary widely with respect to ease of excavation, but each variety has its own set of characteristics developed at the time of its deposition and altered by its post-depositional history. The origin of the deposit—whether alluvial, marine, lacustrine, glacial, or aeolian—very largely controls the nature of the unconsolidated sedimentary deposits as well as the original nature of the sedimentary rocks. The latter, however, may be deformed, metamorphosed, and weathered so as to be more difficult to predict where not exposed. It is the task of the geologist to learn the geologic history of such a rock, and to base his prediction of its properties affecting excavation on that history.

Of the metamorphic rocks, those formed by regional deformation or by load metamorphism at great depth may be expected to be uniform over comparatively great distances laterally, but in some cases they may change abruptly with depth. Such changes at depth are the result of original variations in the rock before it was metamorphosed, and they are predictable to a certain extent. Rocks of contact metamorphic origin are not to be expected to occur over large areas and may be found to grade into their unmetamorphosed equivalents within short distances.

In a study of excavation problems erosional, glacial, sedimentary, igneous, deformational, and weathering histories should all be considered, since the present condition of the materials to be excavated is entirely dependent upon them.

Prevention and Correction of Landslides

Treatment of landslide problems may be divided into two categories, preventive and curative. All too often the opportunities to apply comparatively inexpensive preventive measures are passed by during highway construction, and as a result there occur traffic-disrupting, and sometimes catastrophic, slides that could have been prevented had the proper precautions been taken. The necessary curative measures are generally costly and are seldom completely effective.

In laying out the route for a new highway it is important to watch for places where the probability of landslides is great, and such places should be avoided in locating the highway. Where economic or other factors make it necessary to build a road in an area of probable landslide occurrence, careful preliminary surveys should be made in order to plan proper preventive measures. The preliminary survey must first determine which areas are likely to be subject to landslides. It should then discover the nature of the possible slides and show the factors, geologic, climatic, and man-made, which will initiate rock or soil movement. The investigators should be thoroughly familiar with geology and its rôle in landslide occurrence.

The surface of the earth is constantly undergoing changes in shape. Most of these changes involve the transfer of enormous quantities of material from the highlands to the lowlands and to the sea by erosion and deposition. Quantitatively less important but of far greater direct interest to the engineer are those changes in shape which involve landslides. "Mass movement of earth or rock on hill slopes, at an appreciable rate of speed," will be used as the meaning of the term "landslide"—a definition more inclusive than that generally accepted by geologists but well suited to engineering purposes. The rocks and soil of the earth's crust are under the influence of gravity, so that even in only slightly hilly terrane they are in an unstable state of equilibrium. Highway construction through blasting, altering natural drainage, and transferring earth materials from their normal resting places to new positions upsets temporary stability and initiates many landslides.

Old landslides can usually be recognized by their characteristic irregular, hummock-and-hollow topography in the debris situated at the base of slopes on which the scars of the slides usually may be seen. Where no characteristic topography or landslide scar is detectable, old slides are sometimes indicated by tilting trees or trees with tilted bases and vertical tops.

Landslides may involve enormous masses of solid rock broken from steep faces under their own great weight, or they may involve only loose surficial detritus. Rockfalls, not properly classed as landslides, comprise small quantities of solid rock precipitated from steep slopes. The movement may be in the form of mass sliding or slipping, or it may be in the nature of a flow. In any case it will be conditioned by some or all of these features: (*a*) Presence of clay or decomposable shale, (*b*) underground water, (*c*) topography, (*d*) stratigraphy, (*e*) structure, (*f*) joints, (*g*) foundation, and (*h*) shape of rock particles.

The sliding of rock strata along lines of stratification, sliding of surficial detritus over rock foundation, and flow of detrital masses are all facilitated by the presence of clay or shale which softens when wet. Decomposable shale where overlain in a cut or cliff by rocks more resistant to weathering often causes slides by weathering and sloughing, allowing the overhanging rock to break and slide of its own weight.

Abundant underground water decreases the stability of hillside accumulations of soil and facilitates slides, particularly where clay is present.

A topography of great relief, of course, is most subject to slides, but gentle slopes also are the sites of troublesome slides under some conditions.

A stratigraphic relationship frequently causing trouble is that in which pervious rocks overlie impervious strata, especially where the impervious layer is clay. Percolating water, on reaching the impervious layer, is stopped in its downward movement and is forced to flow laterally to an outlet. Where the outlet is a cliff, slides are quite likely to occur along the upper surface of the water-soaked clay layer.

Unjointed igneous rocks are generally capable of standing vertically without sliding, and hard, unjointed, horizontally stratified rocks are relatively stable, but rock strata dipping toward a steep face are subject to slippage along the planes of stratification. Slides in such rocks seldom give any warning previous to the time of their occurrence. Jointing weakens any rock, and jointed rocks already weakened by stratification are especially likely to cause slides. Rock masses often slip along old fault planes where movement previously occurred.

The angle of repose for rounded rocks is less than that for angular rocks which offer more resistance to slippage and flowage; therefore it seems logical to assume that detritus high in rounded rock and sand grains is less stable and more likely to slide than angular detritus. This assumption has been borne out by the observations of G. E. Ladd (20), Associate Economic Geologist, PRA.

Different combinations of geological, climatic, and man-made conditions can produce slides of quite different natures. Preventive measures should be planned on a basis of a slide's causes, and those causes can usually be discovered by geologic investigation and recognition of the type of slide. The following landslide classification is a modified form of one proposed in 1927 by Mr. Ladd:

(*a*) Slope adjustment of surficial material;
(*b*) Rock strata slides;
(*c*) Rockfalls; and
(*d*) Artificial fill slides.

Slope Adjustment of Surficial Material.—This very common type of slide may involve either flowage or slippage or both. Individual slides vary in size from very small to several thousand feet in length and width. Movement is sometimes rapid and complete within a few minutes or hours, but more often it is slow and intermittent and continues for years. Rate of movement is increased by steep slopes, by the lubrication furnished by clay, and by high water content. Also contributory to sliding are rounded rock and sand grains in the detritus. A smooth, lubricated rock foundation below detritus in many cases forms the base on which slippage takes place, but flowage is comparatively independent of foundation conditions.

Rock Strata Slides.—Rock strata slides range in magnitude from small slips in road cuts to huge, sometimes disastrous slides in which millions of tons of rock are dislodged. An example of the latter is the most disastrous slide in North America, which on April 28, 1903, covered more than a mile of highway and killed more than 70 people in the town of Frank, Alberta, Canada. The town was at the base of the steep face of Turtle Mountain, composed of a great thickness of steeply dipping Paleozoic limestone overlying softer cretaceous sandstone and shale. The limestone was highly jointed at right angles to the bedding, and it was along this series of joints that the failure occurred. The

joint cracks weakened the rock; coal mining in the cretaceous rocks near the base of the mountain may have weakened its foundation; and severe freezing and thawing of water in joints in the rock probably furnished the force which made the entire side of the mountain tear loose and crash down into and across the valley at its base.

Rock strata slides occur most commonly along highways where construction or stream erosion has left steep faces of highly jointed stratified rocks or where it has cut across the stratification of rocks dipping toward the face. Combinations of jointing and alternating hard and soft beds are especially dangerous.

Rockfalls.—Rockfalls are seldom individually large, but on account of their wide and frequent occurrence they constitute a persistent problem in highway maintenance. They consist of a more or less steady dribble of rock fragments which increases during wet or freezing and thawing seasons. Hard, jointed rocks are especially subject to rockfalls, and any rock face at the base of which a large talus slope has accumulated should be avoided if possible.

Artificial Fill Slides.—Slides in artificial fills are by far the most prevalent, and they are the source in some states of the most serious highway problems to be met. Sliding conditions arise from the use of improper design, unsuitable materials, poor compaction, unfavorable water conditions, and weak foundations. Test procedures for the purpose of recognizing these slide-causing conditions have been developed and are used to a considerable degree in many states.

The State Geological Survey of Illinois has investigated several particularly troublesome landslides near Peoria (21). One slide caused a 5-ft movement out of line of a 200-ft slab of the highway along the west side of Illinois Valley. Sliding was believed to occur in an impervious clay zone kept wet by seepages from an overlying porous sandstone stratum. Drainage was suggested as the only cure for the slides.

In 1935 W. C. Morse reported (22) a saving of perhaps $100,000 to the State of Mississippi and the nation resulting from his condemnation of a site for a new bridge to cross the Yazoo River at Yazoo City. The proposed site was such that the eastern highway approach to the bridge would have had to be built for from a half mile to a mile over landslide terrane which was so old as to mask its true identity and accordingly might not have been recognized as such had not a careful examination been made.

An excellent discussion of landslide preventive measures by Mr. Ladd (23) in 1928 includes statements of costs and effectiveness of various methods. A most important conclusion resulting from his study of slides in Ohio and West Virginia was that, in most cases, retaining structures, seldom entirely effective, should be replaced by surface and underground drainage systems to keep moisture below the critical point at which rock masses are unstable.

Competency of Materials for Bridge Foundations

A bridge of any but the smallest size represents a considerable investment. Consequently its location should be considered as permanent, or at least as permanent as the location of the highway of which it is a part. To be permanent a bridge should be designed to meet changing natural and man-made conditions as well as present conditions. It has been said that a chain is no

stronger than its weakest link, and this general thesis is certainly applicable to bridge design. The superstructure of a bridge can be no more sound than its piers and abutments, whose soundness in turn is dependent upon their design and upon the nature of the rock or soil foundations on which they rest (Fig. 2).

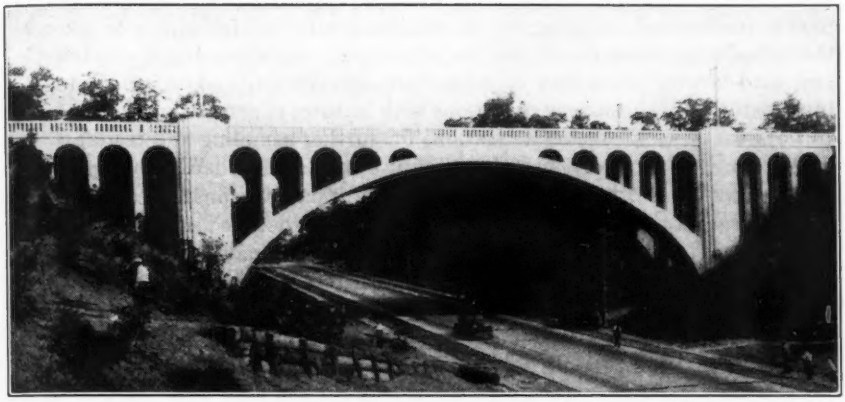

FIG. 2.—THE SUPERSTRUCTURE OF A BRIDGE, ITS PIERS AND ABUTMENTS, CAN BE NO MORE SOUND THAN THE ROCK OR SOIL FOUNDATIONS ON WHICH THEY REST

The weight of the bridge and its traffic load must ultimately be supported by the earth foundation, making it imperative that bearing capacity of the foundation be known so that proper design and spacing of piers can be determined.

Ordinarily economic factors combined with limitations set by the highway route location are the first considerations in selection of bridge sites; foundation materials usually are not considered as limiting factors, the assumption being that the design of the piers can be made to fit the foundation conditions. For this reason bridge sites selected sometimes present difficult foundation problems which require unusually detailed investigation preliminary to construction. Precise information as to the strength, attitude, and water-bearing characteristics of local materials should be obtained for all bridge sites.

Exploratory methods to determine these characters may be classed generally as geological, geophysical, and mechanical sampling. A purely geological exploration of a proposed bridge site is seldom productive of enough detailed and precise data to be sufficient in itself, but a field study of a given site combined with a knowledge of local and general geology should be the basis for rather reliable estimates of probable foundation conditions and should precede choice of more detailed exploration methods. The geological investigation is of particular value in predicting such conditions as depth; structure; and lateral extent of sand, gravel, or soil overlying bedrock.

More precise information of this kind may be obtained through the use of geophysical test methods of several types, the most applicable to engineering work of this kind being the earth-resistivity (electrical) method and the seismic method. A knowledge of geology, especially of stratigraphy and of geologic structures, is indispensable in interpreting the results of any geophysical test.

Taking samples of foundation materials by one of many mechanical means is the most commonly employed exploratory method; the advantage being that in this way subsurface materials can actually be seen and examined, and, when undisturbed samples are taken, tests of crushing and shearing strengths, as well as other physical property tests, can be made. The geologist's sphere of usefulness in mechanical sampling lies in examining samples; in helping to interpret the samples in terms of dimensions, structures, water-bearing characteristics, and load-bearing capacities of subsurface formations; and in combining his interpretation of subsurface conditions with features of general and local geology to form the basis for recommendations for further sampling and testing.

Although in most cases the location of bridge sites is determined primarily by considerations other than foundation conditions, the final selection of sites for the largest bridges frequently is made only after detailed sampling programs have revealed the location of the most satisfactory foundation conditions. In this respect, the San Francisco-Oakland Bridge (San Francisco, Calif.), built at a cost of more than $73,000,000, and one of the largest bridge projects ever undertaken, stands as a monument to the worth of careful preliminary foundation investigations. Before the design or the location of the bridge was selected, the engineers decided on a program of borings and soil tests, the results of which would guide them in selecting (*a*) the most desirable location for the center line of the bridge, (*b*) the best location for individual piers, and (*c*) a logical basis for the design of the piers (24). Samples obtained by several methods were tested mainly in laboratories at the University of California, at Berkeley, and center-line location, pier location, pier design, and construction methods were made to conform to the conditions indicated by the tests.

The too frequent occurrence of bridge failures caused by inadequate foundations indicates that thorough foundation investigations should be conducted for the sites proposed for all bridges, large or small. The following example of a bridge failure should serve to emphasize this point.

In 1912 a $80,000 bridge was built in a west coast city to span a deep cut through which a transverse street passed. The excavation for the cut was entirely in horizontally, thin-bedded, blue clay. Recognizing the unstable nature of such material in the steep banks of the cut, the engineers designed the 95-ft steel cantilever arms and the 171-ft simple steel span so that the arms were strong enough to carry their load without support at their outer ends. The piers supporting the outer ends accordingly were not anchored to the superstructure. The advisability of such a design was proved when in 1915 a slide at the north end of the bridge carried away the piers supporting that cantilever arm without damaging the bridge. Thus, recognition of unstable foundations was the basis for effective superstructure design to meet those conditions; the inadequacy lay in not using the same basis for design of the piers. This was proved on February 26, 1917, when a slide caused the failure of one of the north main piers, badly buckled the lower chords in the south cantilever arm, and caused the collapse of about 150 ft of the timber approach on the south. More soundly anchored piers of proper design very likely would have prevented this bridge failure.

Bridge foundation investigations are conducted regularly by the state geological surveys of several states. Notable in this connection is the work of the Michigan Geological Survey Division in examining the core samples obtained preliminary to plans for a "Straits Bridge" to cross the five-mile-wide Straits of Mackinac between Mackinaw City, Mich., and St. Ignace, Mich.

Desirability of Bridge Sites and Road Sites with Respect to Possible Changes of Stream Channels

Permanency of bridge piers and abutments is dependent not only upon design of the structures and load-bearing capacity of the foundation, but also, to a considerable extent, upon the permanence of the stream channel at the bridge site. The shifting or meandering of a stream channel may dictate long spans and deep piers across the entire width of the flood-stage channel even though the normal channel be only a small fraction of that width. In most instances meandering streams operate to increase maintenance costs rather than first cost of construction. In any case, a careful study should be made for bridge sites on meandering streams, and conditions conducive to the permanence of a channel should be a deciding factor in the final selection of the crossing. The yearly destruction of many miles of highway by shifting stream channels indicates that more emphasis also might well be put on locating highways with more regard to possible channel changes.

Because most changes in the shape of the earth's surface resulting from stream erosion take place so very slowly, people often fail to recognize the tremendous erosive power of most streams. The effects of erosion are emphatically impressed upon the mind of the observer who, standing upon the brink of the thousands of feet deep and miles wide Grand Canyon of the Colorado, sees the muddy river still actively eroding a deeper channel.

The development of the valleys of all rivers, including that of the Colorado, passes through a series of stages (25) recognizable by the consequent topography. During the stage of youth streams have steep gradients and steep sided V-shaped valleys, and they erode rapidly, dominantly downward. During successively later stages the streams' gradients decrease, valleys widen and become less steep sided, and downcutting is largely replaced by lateral cutting. The ultimate development is the peneplain, where erosion has virtually ceased.

Climate and rock types and structures act to modify the effects of erosion. In arid regions much or most of the drainage is interior; that is, streams flow into closed basins and do not connect with the sea. Thus, all material eroded by a stream in such a region is redeposited on land, usually directly in the bed of the stream. Highways in arid regions, where streams flow only during infrequent storms, must be built with due consideration not only to the erosive power of the streams but also to the inevitable and sometimes extensive deposits of sand, gravel, boulders, and mud left in the wake of the flood waters.

Erosion in humid regions is less erratic, and its effects are more easily predicted. In the youth stage erosion is dominantly downward and is most rapid where velocity of current is greatest, at rapids and at constructions of the waterway. Rapids, as a result of accelerated stream bed erosion under them, move

upstream, deepening the channel as they go. During later stages lateral cutting widens valleys and allows flood plains to form. Streams near base level on broad flood plains are subject to comparatively rapid lateral shifts of channel, especially when the rivers are at flood stage. Lateral erosion is greatest on the outside bank at a curve in a stream. Consequently, the broad loops of a meandering stream tend to migrate bodily downstream.

Hardness and chemical reactivity of rocks have obvious effects upon erosion. In regions of sedimentary rocks the tendency is for the hard sandstone strata to resist erosion and form ridges while the somewhat softer and much more soluble limestone beds and the soft shale beds, because they are more easily eroded generally, are the rocks in which the valleys form. In arid regions, where there is little opportunity for solution, limestone is resistant to erosion and is commonly a cliff-forming rock.

Rock structures act to control erosion and in so doing they exert considerable control over stream patterns. The structural features most effective in guiding erosion and most essential to recognize in a bridge-site survey are folds, faults, and joints. Faults and joints provide planes of weakness along which erosion is easily accomplished. Folding governs stream pattern by forcing parallel bands of hard and soft rocks into flexures so that upon erosion the tilted hard strata are separated by nearly parallel softer strata.

The effects upon shifting of streams attributable to stage of erosion and to modifications imposed by climatic and rock structural conditions are recognizable in advance through geologic investigation. Such investigation should precede final selection of every bridge site. In the case to be described briefly preliminary geologic investigation very probably would have condemned the bridge site chosen as liable to dangerous changes of channel, a condition proved to be true after the bridge was built.

In September, 1914, a 280-ft through steel span highway bridge was built across a certain river in the State of Washington. It was built by the county and state at a cost of $22,818. The north end of the bridge was anchored to solid rock, but the south abutment rested on river gravel and sand. High water in 1920 shifted the channel of the river and washed out the south approach. The bridge itself was not damaged, so to cross the new channel a second steel span of 240-ft length was built to connect with the first. The second span was completed in November, 1920, at a cost of $39,402. Later high water again shifted the channel farther southward and this time washed out the south approach of the second span. A third steel bridge was connected to the second to cross the newly formed channel. The third span, completed in 1934, was built at the cost of $36,284. Thus, an unfortunate original selection of bridge site was responsible for later construction cost increase of 331% over the cost of the original construction.

Investigation of Proposed Tunnel Sites

Probably in no other phase of highway engineering can the study of geology be more fitly and profitably applied than in locating, designing, and constructing tunnels. Increasingly higher standards for newly constructed highways necessarily result in the construction of more and more tunnels in order to meet the

maximum grade requirements. Each tunnel built presents problems unique to that tunnel alone as well as problems common to all such projects. This fact, in addition to the high cost of tunneling, emphasizes the need of obtaining as much information relative to rock conditions as possible before construction is begun.

As in bridge building, the general location of the highway limits the possible locations of the tunnels to be built, and economic factors further govern tunnel design and construction. As in bridge building, however, geologic conditions play a large part in determining those governing economic factors. After a preliminary survey for a new highway has established the general location of a tunnel, a careful geological investigation should be made in conjunction with a drilling campaign. This investigation should reveal the existing geological conditions at the tunnel site, and those conditions should then be the basis for selection of (*a*) the final location of the tunnel, (*b*) the construction methods used, and (*c*) the design of the tunnel. An example to illustrate how a geologic survey can be of value in the final selection of a tunnel site is the case of a tunnel built obliquely across the line of an active fault. A flow of water in a wide zone of fault breccia and gouge caused trouble by lubricating the soft material and allowing it to drop into the tunnel. A geological examination of the site should have revealed the presence of the fault previous to construction and should have shown the advisability of locating the tunnel so as to avoid the fault.

After a tunnel site has been selected, an investigation should be made to discover as many as possible of the geological features that will affect the method of construction and the design of the tunnel. Unconsolidated materials such as sand, gravel, and mud require tunneling techniques entirely different from those of solid rocks, and variations of the main methods used also depend upon details of rock characters, most of which can be determined in advance of construction. These details of rock characteristics have a bearing upon (*a*) overbreak of rock in excavation beyond the limits specified by the design engineer, (*b*) hazards of construction, and (*c*) necessity of lining the tunnel.

Overbreakage.—It is practically impossible to excavate a tunnel to the exact dimensions called for in the specifications, and the material excavated beyond those dimensions according to most contracts must be moved by the contractor without pay. Accordingly, the extent of overbreakage to be expected is an important consideration. Plutonic igneous rocks, because of their general textural and structural uniformities, usually break closer to the theoretical section than do sedimentary and metamorphic rocks. The foliation and jointing of metamorphic rocks and the jointing and bedding of sedimentary rocks act to control the directions of breakage during excavation, the breakage being easiest along the lines of weakness furnished by these structural features.

Hazards.—The chief hazards in tunneling operations are the possibility of encountering large flows of water, the danger of cave-ins in soft rock and unconsolidated material, and the danger of blocks of rock falling from the roof of tunnels in solid rock. An additional inconvenience frequently met in deep tunnels is increase of rock temperatures at depth. Temperature increase with depth is variable and its causes are not entirely understood, and so cannot be

predicted with any great degree of accuracy. The 12-mile Simplon Tunnel in Europe, with a maximum rock cover of 7,000 ft, encountered rock temperatures as high as 132.8° F.

The greatest likelihood of encountering large flows of water is found in tunnels in alluvial deposits. Permeable sediments of any kind may carry large flows, as might also permeable extrusive igneous rocks; but large flows of water are not to be expected in intrusive igneous rocks except along faults or joint planes.

Cave-ins and rockfalls are conditioned by the degree of consolidation, the presence of clay or soft shale, vertical and lateral earth pressures, underground water, and textural and structural features of rock.

Necessity of Lining.—The walls and roof of a highway tunnel must be competent to stand without danger of failure for the life of the tunnel. If rock is such as to be hazardous during construction, it probably will require an artificial lining to make a safe tunnel for traffic; but, just because a rock does not give trouble during construction, that is no assurance that it will remain safe and not require lining. Long exposure to air and water causes some rocks to expand and causes others to decompose and lose their strength. Anhydrite and some clays and shales are especially subject to failure in this manner and must be treated accordingly.

Accurate and complete records of the geological data collected prior to, during, and after construction should be kept for every tunnel built. In this way a valuable collection of information may be made for future use.

Investigation and Prediction of Subsurface Water Conditions and Location of Underground Water Supplies

Subsurface water, an entire branch of geologic study, can be only barely mentioned. The subject has been treated comprehensively by C. F. Tolman (26). Additional information may be had from the publications of the Water Supply Branch of the United States Geological Survey, two papers by O. E. Meinzer (27)(28) being especially valuable.

Subsurface water may be divided into vadose water and ground water. The line of demarcation between the two is the water table, the upper surface of the zone of saturation in which ground water fills all the interstices in the rock. Vadose water is that subsurface water which lies above the water table, in the zone of aeration. Movements of ground water are predominantly lateral, whereas movements of vadose water in the zone of aeration above the water table are predominantly downward. Upward movements of vadose water are also of some interest to highway engineers, especially in connection with frost-heave control as previously discussed. Subsurface water has a further important bearing on previously discussed highway problems encountered in open excavations and fills, tunnels, landslides, and bridge foundations. Except in frost-heave problems, vadose water has little effect upon highway projects; the trouble-causing water in most instances is that encountered when highway construction penetrates below the water table.

State highway maintenance stations require supplies of potable water, and many highway construction and maintenance operations require large quanti-

ties of water. In some instances where surface supplies are not available this involves locating and developing ground-water sources, and locating a sufficient supply as near as possible to the surface may be quite an accomplishment. A man versed in geology should be competent to make a scientific prediction, within limits, of quantity available, rate of flow, and depth to satisfactory supplies of water in any given area. The geologist's prediction is not infallible, of course, for the reason that it is made on at least three extremely variable bases, any or all of which may not be entirely dependable in every instance: (*a*) Past experience of others; (*b*) the individual's education and experience; and (*c*) his interpretation of subsurface rock conditions.

Movements of subsurface water are controlled by geologic conditions, chief of which are character, distribution, and structure of the rocks enclosing the water. Areal and vertical extent of a rock formation; details of its general structural features such as abundance, size, and attitude of stratification, joints, and faults; and textural details governed by size, shape, assortment, and arrangement of individual rock grains—are all factors governing subsurface water conditions and all reflect the complex geologic history through which the rock has passed. In general, well sorted, unconsolidated, clean gravel has the highest water-yielding capacity of the rock formations. Other rocks that yield much water, under favorable conditions, are sand, sandstone, limestone, and basalt. Poor water-bearers are hard, unjointed igneous and metamorphic rocks, and most unproductive of all are the clays, which although porous are nearly impervious to water.

The state geologists of several states have been given the responsibility of locating wells to supply highway uses. The Florida Geological Survey suggests the locations for water wells, identifies formations and water horizons, and makes geologic well logs for the state's road camps. The Geological Survey Division of Michigan assists the Michigan Highway Department in the development of water supplies at its roadside parks, and also locates new wells for home sites where homes are necessarily moved as the result of rerouting or widening of highways.

Evaluating Mineral Lands for Highway Purposes

The right-of-way division of a state highway department has the task of securing by purchase, or where necessary by condemnation, rights of way, pit sites, and stock-pile sites as needed. Increased activity in improving old roads and building new ones in late years has greatly increased the volume and complexity of the work of the legal staffs of those divisions, and the expert testimony of economic geologists has been of assistance in a number of situations where mineral lands have been acquired for highway construction.

Owners of mineral lands are prone to overestimate the value of their own property; likewise, people unfamiliar with mining enterprises tend to underestimate the value of such property, although in some instances they have been duped by high-pressure salesmanship into accepting unwarrantedly high values set by promoters for practically worthless property. The unbiased consultory advice of a geologist could be a real aid to a right-of-way legal staff confronted with such situations.

A second use for geologic advice in evaluating mineral lands arises in connection with the construction of mine-to-market roads. The setup in the State of Washington will serve to illustrate this point.

In 1939 an act was passed creating a Mine-to-Market Road Commission, consisting of three members—the Director of Conservation and Development, chairman; the Director of Highways; and the Executive Officer of the Washington State Planning Council—the purpose of which was to act on all matters pertaining to the establishment, location, construction, and financing of mine-to-market road projects. A project can be initiated under this act by a petition which states the known facts pertaining to the occurrence of mineral deposits in the area to be served, the extent of explorations and development, and approximate length, termini, and route of the proposed road. All petitions are investigated by members of the commission, and the properties to be made accessible by the proposed roads are examined by geologists of the Department of Conservation and Development to check the statements made in the petition and to determine the worth of the proposed roads.

Witness in Litigation

Because highway engineering involves earth materials in most of its phases, it is not surprising that a large number of the cases brought to court by or against highway organizations arise from disputes concerning those earth materials. Likewise, it is not surprising that geologists have been called in on many such cases to give expert testimony on geologic matters involved. In passing it might be said that had the geologist's advice been asked earlier many court actions could have been prevented by eliminating the points of contention.

A court case in which geologists' testimony played a significant part was that in which a railroad sued a state for disruption of traffic and repair of immediate damage, and for destruction of stability along 700 ft of filled grade. The damage, actual and claimed, was caused by an avalanche of rock initiated by a large dynamite blast shot on November 22, 1927, by a contractor building a part of a state highway. The estimated 165,000 cu yd of rock removed by the blast was precipitated from the cliff, down a talus slope at its base, across a muck-filled valley for a distance of about 1,500 ft to and over a section of a long railroad embankment in the valley. A total of 700 ft of track was destroyed. The railroad asked for the $20,000 on the basis of their claim that not only the tracks but also the entire rock fill in the embankment was carried away by the movement. The plaintiff's thesis, substantiated by the testimony of a well-known geologist, was that the movement was in the nature of a slide or flow of a mixture of rock and soil talus and of soil in the valley, the effect of which was to push the rock fill out into the river beyond the embankment, leaving in its place a heteorogeneous mixture composed 35% of rock and 65% of muck and silt decidedly inferior to the original rock as an embankment material.

In opposition to this was the state's contention, as presented by the testimony of Harold E. Culver, State Geologist, that the movement was not in the nature of flowing or sliding of blasted rock, talus, and valley fill, but rolling and tumbling of blasted rock only, the effect of which was to fill a part of the valley

and to overflow the embankment taking part of the track with it, but not disturbing the stability of the embankment itself. The evidence presented in Mr. Culver's testimony was influential in the court's decision to disallow the railroad's $20,000 claim.

Conclusion

The task of the engineer is to overcome, by his ingenuity, the obstacles presented by nature. His works are becoming increasingly more complex and of greater magnitude and consequently require the use of highly technical information in an increasingly wider range of fields. It has become impossible for any one man to command a knowledge of much more than the bare fundamentals of all of the many highly specialized lines of endeavor involved in the successful completion of many engineering projects. This increased complexity in all fields has given rise to a so-called age of specialization, in which specially trained men in many fields combine their efforts to produce the best possible results. Highway engineering is sufficiently broad to include a great number of varied kinds of work and so has involved specialization in many fields, among them that of geology, dealing with the earth materials and the forces that act upon them—features which in one way or another enter into almost every phase of highway engineering.

Some of the services already rendered to engineering by geologists have been stated in this paper, with the hope that in this way engineers and geologists alike may be helped to recognize more frequently those situations in which the geologist may be of assistance to the highway engineer.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT

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SYNOPSIS

The Boulder Canyon Project, after a generation of investigation, planning, and political debates, progressed to actual construction in 1930. First commercial power production occurred in October, 1936, with additional generating units still being installed as of 1944. Since June, 1943, the power output has exceeded 1,000,000 kw, the first time in history for a single power plant; so the project's record of breaking records is continued. The ultimate capacity of the plant is announced as 1,332,300 kw.

The great dam, 726 ft high, controls the flow of the Lower Colorado River on the Arizona-Nevada boundary and creates a storage reservoir of 32,000,000 acre-ft capacity, called Lake Mead. It was built and is owned and operated by the United States because of the complexity of its interstate and international problems, which were the more involved due to its being the pioneer in the field of large multiple-purpose hydro-power projects. In addition to improving navigation (for judicial purposes, at least), controlling floods, and regulating the river flow for irrigation, domestic, and municipal uses, the power production is sufficient to amortize the project's construction cost and guarantee payments to Arizona and Nevada in lieu of taxes, plus contributions to a basin development fund. Silt control and regulation of water quality are important incidental results.

The power rates were adjusted in 1941, on an amortization instead of the previous competitive market rate basis. The reasons for this change are explained in this paper, with detailed comparison of the two plans as regards allocation of construction costs.

The Boulder Canyon Project has now gone to war in a big way, of course;

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May 1, 1944.**

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but, even before the war boom and the earlier defensive rush period, its success was assured, with power output and cost repayments running far ahead of advance estimates. As of 1944, however, one near-by metallurgical (magnesium) plant requires 200,000 kw and every available kilowatt-hour is devoted to the war industries of Southern California. Stored water is also pumped for these industries and the municipalities around them through the Colorado River Aqueduct.

Boulder Canyon Project Act

The Congress of the United States, by the terms of the "Boulder Canyon Project Act," approved on December 21, 1928, authorized the Secretary of the Interior

> "* * * to construct, operate and maintain a dam and incidental works in the main stream of the Colorado River at Black Canyon or Boulder Canyon * * * for the purpose of controlling the floods, improving navigation and regulating the flow of the Colorado River, providing for storage and for the delivery of the stored water thereof for reclamation of public lands and other beneficial uses exclusively within the United States, and for the generation of electrical energy as a means of making the project herein authorized a self-supporting and financially solvent undertaking; * * * also to construct and equip, operate and maintain at or near said dam, or cause to be constructed, a complete plant and incidental structures suitable for the fullest economic development of electrical energy from the water discharged by said reservoir; * * *."

As indicated by this preamble to the Act, the solution of the Colorado River problems, as plans were gradually formulated after investigations over a period of many years, necessarily required a multiple-purpose project. This was perceived long before such a conveniently descriptive term became so widely used and accepted in discussions of engineering economics. Largely to this diversity of objectives, and the serious international as well as interstate complications that were involved, was due the final authorization for the construction by the Bureau of Reclamation, U. S. Department of the Interior, with federal appropriations. Indeed, the Boulder Canyon Project is generally recognized as having been the pioneer in the field of major multiple-purpose hydro-power developments.

In addition to the numerous general purposes as listed, the removal of Colorado River silt (long infamous as often "too thick to flow even though too thin to plow") has proved of vital importance, for, without such silt control, neither power installations nor water diversions could be considered as economically and permanently feasible. (The reservoir has an estimated 300-yr life, as a minimum, which will be indefinitely prolonged by the construction of numerous planned upstream dams.) Likewise, the improvement of the quality of Colorado River water by decrease of its low-flow salinity and hardness, due to storage and mixture of the previously wasted and relatively pure flood flows, was immediately of real value. Such quality improvement has become even

more evident as the first filling of Lake Mead (Arizona-Nevada) and two years (1941 and 1942) of large discharges (fortunately chiefly through the gates rather than from the purer surface stratum over the spillways) have continued the leaching of the gypsum-mixed sands included in the desert reservoir's flooded area. This improvement is only temporarily retarded by the 1943 low inflow of but 70% of normal. Even the improvement of navigation, generally dismissed as merely a legal fiction, has permitted the barging of ores, previously inaccessible, on a commercial scale, as well as a recreational patronage for fishing, bathing, sailing, and motor boating, to a most unexpected extent. Visitors to the power plant until it was closed to the public on December 8, 1941, averaged more than 1,000 per day for a total of nearly two million (Fig. 1), besides still larger numbers who merely admired the dam and scenic vicinity.

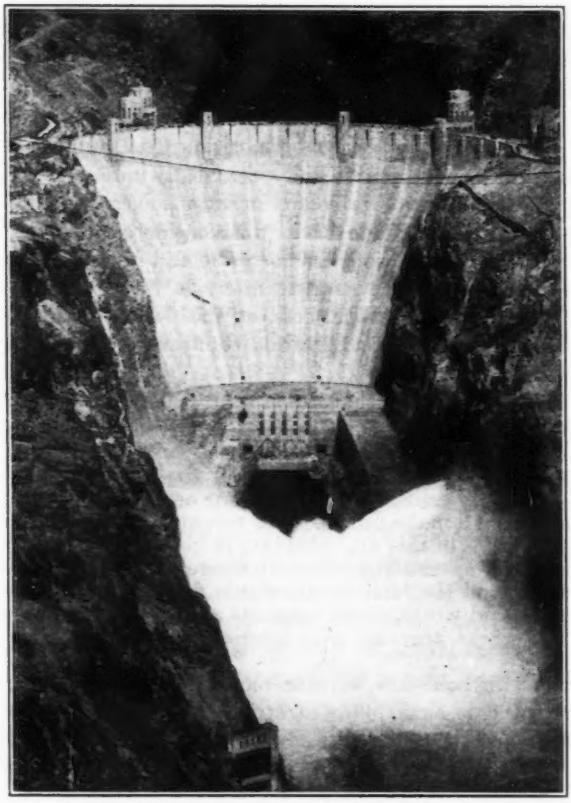


FIG. 1.—VISITORS TO THE POWER PLANT, UNTIL IT WAS CLOSED TO THE PUBLIC ON DECEMBER 8, 1941, AVERAGED MORE THAN 1,000 PER DAY

Evidently the Boulder Canyon Project has really accomplished a multiplicity of purposes.

A draft of the Swing-Johnson bill, as the project Act was generally known, was introduced in Congress as early as 1923, but enactment was delayed during

five years of nearly continuous and heated debate over the basic policy of governmental ownership and construction of power dams. The question of a single-purpose versus multiple-purpose project also played a part in the controversy, as some opponents of government control of the high power dam favored instead a separate, relatively low dam at another site to be built by the United States solely for flood control, the immediate need for which seemed unquestioned. This would have left the canyon section of the Colorado River for private power development, probably taking the form of a series of several relatively low dams. The apparent economy of a multiple-purpose dam was a deciding factor in finally securing favorable Congressional action, as advocates of government ownership asserted (correctly, as is now proved) that power and other revenues would be amply sufficient to repay the cost of the project with interest. That the approval of the Boulder Canyon Project Act established a notable and important precedent is now very evident. Although the Bureau of Reclamation had previously constructed about fifteen power plants, incidental to its irrigation projects, their combined cost was less than 1% of the authorized investment for the Boulder Canyon Project. Few of these small installations had encountered serious opposition from public utilities, as they had offered little or no competition and were mostly in remote and undeveloped localities, at the time unprofitable for private power installations. On the other hand, the Lower Colorado River was considered a choice prize, for the control of which the competition was energetic and prolonged. Congressional and public sentiment finally swung definitely in favor of governmental construction of such hydro-power projects, as the decade following the passage of the Boulder Canyon Project Act has confirmed, with its series of great federal dams.

In line with the arguments that the project would be self-liquidating in fact, a definite plan of cost repayment was written into the Act (section 4-b)—

> "Before any money is appropriated for the construction of said dam or power plant, or any construction work done or contracted for, the Secretary of the Interior shall make provision for revenues by contract, in accordance with the provisions of this act, adequate in his judgment to insure payment of all expenses of operation and maintenance of said works incurred by the United States, and the repayment, within fifty years from the date of the completion of said works, of all amounts advanced to the fund under sect. 2(b) for such works, together with interest thereon * * * (at 4%)."

This requirement for bona-fide, enforceable repayment contracts, covering the estimated project cost, was followed strictly by the then Secretary of the Interior, Ray Lyman Wilbur, M.D., before President Herbert Hoover, Hon. M. Am. Soc. C. E., approved the first appropriation for initiating construction work on July 3, 1930.

Such an advance guarantee of cost repayment with interest made the Boulder Canyon Project unique among national developments, a record unshared as yet but only emphasized as to its rarity by the cases of numerous subsequent and comparable power dams constructed under federal auspices. The record is more notable in view of the rigid interpretation and enforcement given the repayment clauses of the Act, quite at variance with the long-

established repayment practices on irrigation and other government projects (leniency having been often allowed, generally for good and sufficient reasons). The several contracts negotiated with the Los Angeles (Calif.) City Power Bureau, the Southern California Edison Company, and The Metropolitan Water District of Southern California, which were later supplemented by contracts with various other power distributors, represented probably the largest power transaction to date. They called for then-estimated payments over a fifty-year period of about one-half billion dollars for merely the primary project, not including costs of long transmission lines and complex distribution facilities. The enormity of such repayment promises and the seriousness and importance to all concerned of these contracts are now difficult to realize under subsequent changed conditions of more easily promoted projects as items of progressively greater relief, defense, and war budgets. The support of the Southern California Edison Company, in underwriting a large portion of the cost of this governmental and municipally-favored development, is especially notable, as inaugurating a period of enlightened public relationship and ending, regionally at least, a protracted and "totalitarian" war of uneconomic competition.

Colorado River Dam Fund

Early drafts of the Swing-Johnson bill provided for financing by a project bond issue but the Act as finally passed relied instead on normal Congressional appropriations. Patterned to some extent after the regular Reclamation fund, a special fund was established, designated as the Colorado River Dam Fund, to be available only for carrying out the provisions of the Act. Greatly simplifying the accounting procedure, and cost or repayment auditing reports, it has somewhat the relation to the U. S. Treasury of a subsidiary to a parent corporation. It appears to have served as a "small-scale" precedent for the subsequent depression era's Reconstruction Finance Corporation (as a means of financing public construction) and other government corporations and funds, which have become such a favorite and accepted method of federal administrative and accounting practice.

All advances to the Colorado River Dam Fund from the U. S. Treasury were required to be repaid with 4% interest, but a definite allocation of $25,000,000 was made to flood control to be repaid out of 62.5% of excess revenues, if and when available. If this repayment were not completed within the general fifty-year amortization period, ending May 31, 1987, repayment was to be continued out of the same proportion of the project's net revenues. Thus deferment was permissible for the repayment of the flood-control allocation, if this became necessary, but provision was made for full repayment finally, with 4% compound interest. The amount of this flood-control allocation appears to have been fixed arbitrarily, so far as is now evident, but is approximately the sum that might have been needed for a program of flood control by means of a low dam for this purpose solely, and a system of raised and strengthened levees. This solution would have given immediate, but evidently only temporary, relief from flood damages, due to relatively rapid filling with silt of a small reservoir. The federal obligation to furnish some degree of flood control on the Lower Colorado River was recognized, however, and this is probably the

only case to date in which repayment (even though possibly on a deferred basis) of a government flood-control structure, or its allocated share of the construction cost, has ever been required or promised.

In addition to the flood-control allocation, other excess revenues of the Colorado River Dam Fund were allotted as follows:

> "If during the period of amortization the Secretary of the Interior shall receive revenues in excess of the amount necessary to meet the periodical payments to the United States as provided in the contracts executed under this act, then, immediately after the settlement of such periodical payments, he shall pay to the State of Arizona 18¾% of such excess revenues and to the State of Nevada 18¾% of such excess revenues.
>
> "After the repayments to the United States of all money advanced with interest, charges shall be on such basis and the revenues derived therefrom shall be kept in a separate fund to be expended within the Colorado River Basin as may hereafter be prescribed by the Congress."

Records of Congressional Committee hearings indicate that the intent of Congress was that Arizona and Nevada should receive some payments in lieu of taxes which would have been levied if the project had been privately developed. The amount is stated as having been determined on the basis of a fairly parallel case, the leasing of public oil lands, where the rule had become established that the state involved should receive 37% of the net revenue in lieu of taxes. The disposition of the balance of excess revenues, after completion of repayment of U. S. Treasury advances to the fund, was such a remote problem as not to excite immediate interest, so that it was merely left to future action of Congress, though allotted for Colorado River Basin developments if and when available.

Original Repayment Contracts

As repayment contracts were required by the Act in advance of even the beginning of construction, their terms were necessarily based on estimated project costs. Power production had also to be estimated from stream-flow records and reservoir operation studies, at first being called 4,240,000,000 kw-hr annually of firm power initially, subject to some gradual depletion. This amount was taken as 100% in the allocations of power, although a later decision to add 25 ft to the proposed height of the dam (550 ft) yielded a further block of 90,000,000 kw-hr of firm power annually. Applications for the power originally totaled more than three times the estimated production, with the City of Los Angeles and the Southern California Edison Company each asking for all of the available power. Final power allotments were made only after prolonged hearings and many protests and compromises, with eight contracts resulting from twenty-seven applications, some of which were conditional or indefinite, however. The repayment guarantees involved three main contracts:

City of Los Angeles	37%
Metropolitan Water District	36%
Southern California Edison	27%
Available power	100%

The City of Los Angeles underwrote 6% for other municipalities (which finally amounted to 4.0946%), the Southern California Edison Company similarly underwrote 0.9% for each of two other power companies, and the city and the company each were subject to a drawback of 18% by the states of Arizona and Nevada. As the Metropolitan Water District had not at the time held its aqueduct bond election, legal questions were anticipated as to the strict enforceability of its contract. The guaranteed portion of the project cost was therefore finally covered by the City of Los Angeles and Southern California Edison Company contracts for but 64% of the estimated power production.

These contracts were required to provide revenues sufficient in the judgment of the Secretary of the Interior to amortize the project cost, except for the flood-control allocation; but in addition the Act required that the contracts "shall be made with a view to obtaining reasonable returns" as found to be "justified by competitive conditions at distributing points or competitive centers." The latter condition would necessarily have been a determining factor in fixing the power rate even in the absence of such a statutory direction. In this case, the only significant power market was in Southern California, located over large oil deposits, so that the cost of energy provided by oil and gas necessarily fixed the comparative value of Colorado River hydro power, with proper allowance for transmission costs. Studies made for the Secretary of the Interior on this basis indicated a value at the switchboard for the use of the available falling water of about 1.63 mills per kw-hr. Therefore, power allocations were made on the basis of this rate, which the corresponding secondary power rate fixed rather arbitrarily at 0.5 mill per kw-hr.

Original Cost Allocations

With firm contracts executed on April 26, 1930, for the entire power production, and the power rate fixed and agreed to as described, it was widely assumed that an allocation had been arrived at not only for the project cost but also for a dependable surplus, payable to Arizona, Nevada, and the Colorado River Basin Development Fund. Technically, to be strictly accurate, no such definite allocation of costs really had been made, because the Act prescribed that upon the demand of either party to the power contracts, at the end of fifteen years (in 1945) and every ten years thereafter, the power rate should be readjusted as found by the Secretary of the Interior to be justified by changed competitive conditions. In 1930 this factor seemed to mean little, as the first adjustment date was so far into the future, and elaborate, detailed computations and studies were prepared on the assumption of a uniform rate of 1.63 mills per kw-hr. Misleading conclusions resulted and the expenditure of anticipated surplus revenues was budgeted by those interested as if the fund accumulation were certain and automatic. As the national economic depression became more severe and protracted, affecting seriously both power markets and the all-important price of oil, or equivalent cost of gas, it became apparent that under prevailing competitive conditions the power rate had been set higher than was warranted or than could be maintained, in all probability, after the 1945 readjustment. Various alternate studies of anticipated revenues, with

corresponding allocations of cost, were prepared as a basis for discussions of the resulting problems among representatives of the government, the power contractors, and the states interested in excess revenues. Several of these studies that were officially issued by the Bureau of Reclamation and are still of interest, for purposes of comparison, have been tabulated in summary form herein, following a listing of the project costs that serve as a base for the amortization computations.

Cost estimates used in negotiating the original power contracts for the Boulder Canyon Project are derived from a statement[2] by Secretary Wilbur:

Estimated cost (including interest during construction but excluding flood control)	$ 82,674,907
Interest at 4% during 50-yr period, excluding flood control	108,107,007
Interest charges on accumulated deficit	63,973
Estimated operation and maintenance (O & M)	7,262,557
Estimated depreciation	8,875,553
Total requiring "guaranteed" repayment	$206,983,997
Anticipated power revenues at 1.63 miles per kw-hr	
City of Los Angeles	$121,310,549
Southern California Edison Company	88,523,915
Subtotal	$209,834,464
Metropolitan Water District of Southern California	118,031,886
Total estimated falling water revenues	$327,866,350

The distribution of surplus revenue, if and when realized, according to the terms (as then interpreted) of the Boulder Canyon Project Act is:

Repayment of flood-control allocation	$ 25,000,000
Interest at 4% on flood-control allocation	20,981,303
18¾% of surplus to Arizona	22,665,441
18¾% of surplus to Nevada	22,665,441
Balance for Colorado River Basin Development Fund	29,570,168
Total estimated surplus at 1.63-mill rate for fifty years	$120,882,353

In addition to the foregoing totals, power machinery costs were estimated at $17,717,000. It was originally contemplated that this investment would be financed by the power-plant lessees, so that its repayment was not covered in the original power contracts. Therefore, although finally financed with federal funds, along with the remainder of the project, power equipment costs are being repaid over a 50-yr period with interest by means of separate contracts with the generating agents, the City of Los Angeles, and the Southern California Edison Company. Actual project construction costs, as of September, 1940[3]

[2] House Hearing, 71st Cong., 2d Session, Second Deficiency Appropriation Bill, 1930, p. 950.

[3] *Bulletin No. 1*, Boulder Canyon Project Final Reports, Pt. IV, 1941, pp. 26–28; also Exhibit 33, September 14, 1940.

(presented at the Rate Adjustment hearings at Los Angeles) can be summarized in abbreviated form as follows:

Feature	Costs
Dam and Reservoir	
Right of way and mining claims	$ 782,219
Surveys, tests, and examinations (field)	766,335
Stripping canyon walls	627,499
River diversion, cofferdam, dredging	2,348,984
Diversion works (four tunnels)	26,996,604
Main dam and miscellaneous	23,578,724
Spillways	5,868,792
Outlet works	16,337,587
Permanent cableway and landings	607,913
Subtotal	$ 77,914,657
Power Production	
Power-house substructure	$ 5,094,098
Power-house building, water, oil, and air systems	3,726,170
Penstocks	3,330,950
Electrical equipment	13,624,964
Hydraulic, mechanical, and general station equipment	6,620,512
Switchyards	3,265,609
Subtotal	$ 35,662,303
General Plant	
Roads, railroad, construction and concrete plant	$ 2,491,967
Boulder City, Nev., government buildings, telephone system, etc.	2,182,565
Materials and supplies (field)	305,511
Subtotal	4,980,043
Total investment (field)	$118,557,003
Overhead and Undistributed	
Engineering and inspection (Denver, Colo., office)	$ 2,875,341
Concrete and other research	624,431
Colorado River Board and consultants	83,887
Legal, general, printing, furniture, etc. (Denver)	406,602
General and overhead (Washington, D. C., office)	282,076
Secondary investigations, Colorado River Basin	314,416
Subtotal	4,586,753
Total project cost to September, 1940	$123,143,756

Installations of additional power-plant equipment, continuing into 1944, and other permanent work of a minor nature, raised the project cost to approxi-

mately $130,000,000, with the completed cost to be somewhat greater. The most recent available studies, dated February 19, 1941, indicate a final reimbursable cost of $82,878,131 for the new power (falling water) rate determinations, not including generating machinery and the deferred flood-control allocation. This is not exactly comparable with the 1930 estimate of $82,674,907, as the latter includes interest during construction at 4% and also interest during construction at the same rate on the deferred flood-control allocation of $25,000,000. Under the present revised Act and regulations, the latter is considered to be the sum of the first project expenditures instead of being an amount deducted from the construction cost on May 31, 1937, as previously, so that interest during construction on this amount is now omitted, and on the remainder of the cost is computed at a 3% rate. This adjustment for revised interest during construction is estimated as amounting to about $5,000,000 which may be considered the approximate overrun on the cost of the dam and power-plant building, as compared with the 1930 estimates. Most of this increase appears to have resulted from the inclusion of exceptionally thorough and costly research, basin surveys not directly related to project construction, very considerable overhead, and unanticipated expenditures for Boulder City and for guide, scenic, and protection facilities, etc. The actual field construction cost of the dam and power-plant building evidently has exceeded the 1930 estimates but slightly, if at all.

Early rapid progress on the dam was aided by the lowest runoff year (1934) on record, and, with the favorable depression prices of the same period, financial studies made in 1936 (Denver Drawing No. 45-D-10,327 dated September 18, 1936) were allowed to become optimistic to the extent of estimating the reimbursable cost as of May 31, 1937, at $75,927,921. Due apparently to cost accountants being overworked, a considerable lag occurred before the effect of large overhead and indirect costs became fully evident. Annual O & M costs were estimated at but $102,000, compared with $145,000 in 1930 (present studies include O & M costs at $554,000 annually) and, on the other hand, allowances for secondary power and water storage revenues were somewhat more liberal than in either 1930 or at present (1944). The combined effect of these various factors, in addition to the previously mentioned faulty assumption of a 1.63-mill rate for fifty years, was to increase the estimated payments to Arizona and Nevada by 30% and to the Colorado River Basin Development Fund by about 90%, as compared with the quoted 1930 anticipations. Naturally, these larger amounts became fixed in the minds of those interested as being so certainly available within the near future that subsequent smaller but more accurate versions of probable surplus revenues were correspondingly unpopular and dismissed as propaganda.

Beginning in 1937, however, such financial studies were revised to conform to the quoted requirement of the Act (section 5) that payments into the separate Colorado River Basin Development Fund should not be made until completion of all repayments to the United States. Earlier interpretations had assumed that amortization of the dam's cost would cover the permitted 50-yr period, whether such a long time was necessary or not, with surplus funds first amortizing the flood-control allocation and then being paid to the separate fund.

On the corrected basis, however, the federal government share of the surplus was devoted to accelerated project cost amortization, with initial payments into the separate fund being postponed from about 1960 to the very end of the amortization period, 1985 or later, as well as being greatly decreased in amount. The last such study, prepared by the Bureau of Reclamation as of May, 1940, conforming to the original Act but with then-current assumptions for the reimbursable total and estimated O & M costs, and involving varying future power rates, both firm and secondary, after 1945, is available in print.[4] Detailed tabulations are shown for firm power rates of 1.30 and 1.08 mills after 1945, with the secondary rates proportionately decreased. Using identical assumptions and computation methods, similar tabulations have been prepared for 1.63 mills and other future firm rates. These several studies were then brought a step further up to date by corrections for the 1941 estimate of future O & M costs of $554,000 annually (instead of $270,000 annually). They are summarized in Table 1 on a present-value basis, to show the varying allocations

TABLE 1.—ESTIMATED SURPLUS REVENUES AND VARYING ALLOCATIONS[a] IN PERCENTAGE OF ACTUAL-PLUS-DEFERRED INVESTMENT[b]

Firm power rate after 1945 (mills per kw-hr)	SURPLUS		ARIZONA AND NEVADA		DEVELOPMENT FUND[c]		REPAYMENT[d]	
	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of allocation
(*a*) ANNUAL COST OF OPERATION AND MAINTENANCE AS ESTIMATED IN MAY, 1940 ($270,000)								
1.63	45.0	49	17.3	19.0	2.7	3.0	25.0	100
1.555	40.0	44	15.0	16.4	0.0	0.0	25.0	100
1.40	29.7	33	11.1	12.2	0.0	0.0	18.6	74
1.30	22.9	25	8.6	9.4	0.0	0.0	14.3	57
1.163	14.0	15	5.2	5.7	0.0	0.0	8.8	35
1.08	9.0	10	3.4	3.7	0.0	0.0	5.6	22
(*b*) REVISED ANNUAL COST OF OPERATION AND MAINTENANCE AS ESTIMATED IN FEBRUARY, 1941 ($554,000)								
1.63	40.0	44	15.0	16.4	0.0	0.0	25.0	100
1.555	35.0	38	13.2	14.5	0.0	0.0	21.8	87
1.40	24.7	27	9.3	10.2	0.0	0.0	15.4	62
1.30	17.9	20	6.7	7.3	0.0	0.0	11.2	45
1.163	9.0	10	3.4	3.7	0.0	0.0	5.6	22
1.08[e]	9.0	10	3.4	3.7	0.0	0.0	5.6	22

[a] Expressed as present values, at 4% interest rate, as of May 31, 1937. [b] Taken as 91.2 million dollars on a present-value basis as of May 1, 1937. [c] Colorado River Basin Development Fund. [d] Flood-control allocation. [e] With a firm power rate of 1.08 mills, $5,000,000 (present value) of project investment would have been unpaid in a 50-yr period.

of revenues and costs that might have occurred under the terms of the original Act, according as future competitive power conditions more or less severely affected the contracted revenues. Conforming to the May, 1940, O & M estimate, the fixed charges have a present value, computed at 4% as of May 31, 1937, of 100.3 million dollars, including past and deferred investment of 91.2

[4] Hearings on H. R. Doc. No. 9093, Committee on Irrigation and Reclamation, 76th Cong., 3d Session, p. 177.

million dollars in dam and power-plant building (but not hydraulic machinery), plus operation and maintenance, depreciation, and replacements. All of these costs would have been repaid with any firm power rate, after 1945, of 1.08 mills per kw-hr or greater. Correcting the O & M estimate to the 1941 basis, the present value of all such fixed charges is increased to 105.3 million dollars. A firm power rate of at least 1.163 mills per kw-hr is required to amortize this amount within fifty years at 4% interest. The lower rate of 1.08 mills, if applied after 1945, would have repaid only 95% of the reimbursable project cost, in addition to O & M and depreciation charges. The varying amounts of surplus, and its allocation for the several specified purposes for different firm power rates after 1945, are given in Table 1 for comparison.

This tabulation is an extremely condensed summary of voluminous, detailed cost and revenue studies, referred to previously but not included herein because they are now obsolete and of interest only in reviewing the evolution of the present power contracts and rates. A full explanation of their engineering, legal, and accounting complexities would also be most voluminous, and likewise obsolete. In further explanation of Table 1, however, it may be noted that in all cases included, except the first (1.63 mills power rate for fifty years and preliminary estimate of O & M cost), the present value of the hypothetical payments to Arizona and Nevada is 37.5% of the corresponding present value of the surplus, as is specified in the Boulder Canyon Project Act. With the assumptions of the first case, however, the flood-control allocation of $25,000,000 would have been amortized in about thirty-eight years, as a result of the larger revenues. Then for the next nine years the government's 62.5% share of the surplus would have been applied to accelerating the amortization of the project construction cost. During the last three years of the life of the contracts, Arizona and Nevada would not have shared in the surplus (the Act limited their payments to the amortization period), but instead the entire surplus would have been paid into the Colorado River Basin Development Fund, for subsequent appropriation by Congress. The actual disposition of the surplus revenues would thus have been as follows, on a present-value basis:

	Million dollars
Net surplus (1.63 mills rate for fifty years)	45.0
"Surplus" applied to project cost amortization	+3.8
Surplus of last three years for Colorado River Basin Development Fund	−2.7
Portion of surplus shared by Arizona and Nevada	46.1
37.5% to be divided between Arizona and Nevada (or 38.5% of net surplus and 35.5% of full 50-yr surplus)	17.3

As some estimates of the probable firm power rate after 1945, revised on the basis of changed competitive conditions, ranged as low as 1.0 mill per kw-hr, and as this rate would have repaid (within fifty years and at 4% interest rate) only 88% of the investment in the dam and power-plant building, considerable concern arose as to the solvency of the project finances. By the terms of the original Act. the readjustment of the power rate in 1945, and at 10-yr intervals

thereafter, was to be the responsibility of the Secretary of the Interior, as might be justified by competitive conditions. The opinion was frequently expressed that a rate lower than required to amortize the reimbursable portion of the project investment within fifty years was not permitted by the Act; but the legality of this idea was in disagreement and was indeterminate. This result was not improbable, however, as the manner of such a rate redetermination was left vaguely undefined by the Act and would be complex and difficult as well as seriously controversial. Even if the rate could be maintained high enough to insure repayments to the U. S. Treasury, however, the payments to the states and the basin fund were obviously in jeopardy. General recognition of this fact was the cause and basis of the negotiations for a revision of the original Act.

Revenues from Water Storage

Project cost allocations and amortization have been analyzed without special reference to anticipated water revenues because estimates of the latter, as included in the various official financial studies of the Boulder Canyon Project, have amounted to only 2.5% to 3% of the total expected revenues. Furthermore, no adjùstment was required or has been made with respect to water charges, all recent discussions having involved power questions only. Water storage revenues are incidental and relatively minor in importance, in the case of this multiple-purpose project, partly because conditions were so favorable for power development. The high head and large generating units, rated at 82,500 kva, made possible a low power cost; and the large power demand, even with competitive marketing conditions of the depression period, has kept the project self-liquidating. More direct and fundamental factors control the water revenue that can be secured for the project.

The Boulder Canyon Project Act specifically exempted lands in Imperial and Coachella valleys, California, from any water storage charge. The first area is now, and the latter will soon be, supplied by the All-American Canal, also constructed by the Bureau of Reclamation under the authorization of the same Act. The justification for this exemption is that large appropriative water rights of very early priority had been established by such Lower Colorado River projects as Palos Verdes, Imperial, and Yuma valleys. Upstream developments gradually encroached on these prior water rights, causing more and more frequent shortages of irrigation water. These were climaxed by the most severe drought on record in 1934, just a few months before storage began in Lake Mead on February 1, 1935. Such water shortages were largely limited to the Imperial Valley because its diversion point was then below all others on the river, and litigation against junior upstream diversions had seemed unavoidable until the Colorado River Compact was negotiated. Due to these circumstances, the California ratification of the Compact was conditional on the construction by the federal government of a reservoir of at least 20,000,000 acre-ft capacity, at or below Boulder Canyon on the Colorado River.

A considerable portion of the investment in the Boulder Canyon Project may therefore be conceived as a contribution to prevent threatened litigation, and to stabilize past as well as future water rights and diversions in the Upper as truly as in the Lower Colorado River Basin. Except for the effect of such

upstream diversions, now legalized by the Compact and the storage operations of Lake Mead, the natural, unregulated flow of the Colorado River would have been practically sufficient for Imperial Valley irrigation. Present exemption from reservoir storage charges is merely a recognition of this fact.

Other diversions from the Lower Colorado River are not definitely exempt from Boulder Canyon Project storage charges, and the reasonableness of such a charge (fixed at $0.25 per acre-ft) as a contribution toward the repayment of the cost of the project was recognized, for example, in the water contracts for the Colorado River aqueduct of The Metropolitan Water District of Southern California and for the City of San Diego, Calif. Even with the later priority of the water-right appropriation for the aqueduct, however, diversions might have been made during half or more of the time in most years from the river's natural flow, without dependence on storage. The present (1944) charge of $0.25 per acre-ft for all water diverted by the aqueduct, therefore, is in effect, about a $0.50 charge for water really derived from Lake Mead storage. Incidental benefits are obtained as regards desilting and improvement of water quality during non-flood periods, so that the charge on all water diverted was agreed to, it being realized that no accurate or practical separation or determination of natural flow could now be made. A rate of $2.00 per acre-ft was demanded at various times by Arizona representatives, but this was considered to be inequitable and impossibly exorbitant, amounting to a 22% increase in the $8.00 per acre-ft base price of water furnished by the $200,000,000 aqueduct. The rate of $0.25 per acre-ft as determined by the Secretary of the Interior was found justified on a showing that, if the portion of the Colorado River runoff made utilizable for the first time by Lake Mead regulation (about 10,000,000 acre-ft annually) was charged approximately this storage fee, the actual reservoir cost, omitting power production features and the flood-control allocation, could be amortized within fifty years from this revenue source alone, disregarding power revenues. Power revenues are available, of course, amounting (at the average reservoir power head) to $0.73 per acre-ft with the original rate of 1.63 mills per kw-hr, and to $0.52 per kw-hr with the 1941 adjusted rate of 1.163 mills per kw-hr.

On this comparative basis, the water charge does not seem disproportionate to the relative services rendered. It must be emphasized, of course, that the charge is not for the sale of water but rather for storage service only. Prior to 1943, the only contracts for water deliveries other than to the exempt irrigation projects were with The Metropolitan Water District of Southern California and the City of San Diego. Whether a water charge would be made for other projects, proposed or under construction, that will involve diversions from the Lower Colorado River, was not previously evident, as no delivery contracts had been issued for such projects. To be conservative, therefore, all previous financial studies for the Boulder Canyon Project have neglected the possibility of such additional water revenues. In May, 1943, a water contract was announced with the State of Nevada for diversions from Lake Mead, above Boulder Dam, with a charge of $0.50 per acre-ft, including about $0.15 per acre-ft for storage service and $0.35 per acre-ft as compensation for kilowatt-hours lost at Boulder Dam. In present drafts of a proposed water contract

with the State of Arizona (not as yet approved by the Secretary of the Interior) the same charge is fixed for diversions from Lake Mead, and a maximum of $0.25 per acre-ft for downstream diversions. An actual storage charge of $0.15 per acre-ft for such agricultural diversions now seems probable. In view of these recent developments, the project's total water revenues by 1987 may considerably exceed the $7,500,000 previously estimated.

Boulder Canyon Project Adjustment Act

The 50-yr amortization period of the project began on June 1, 1937, with the announcement of the Secretary of the Interior that storage and installed generators were sufficient for the production of 1,250,000,000 kw-hr annually for the first power contractor, the City of Los Angeles. Power had already been sold at the secondary rate, however, amounting to $247,000 under interim contracts. During the construction period of seven years after the execution of the contracts, various events had occurred which forced the conclusion that the 1930 power rates were inequitable and excessive, and could not long be sustained. The competitive value of the hydro power had fallen, as a result of decreases in the cost of fuel and in the capital costs of steam plants, as well as improvements in the art of steam power generation. The record indicated that advances to the construction fund by the U. S. Treasury had been secured at an interest rate averaging about 2.75% and that to charge the project a rate in excess of 3% would involve an unintended and undue profit. The United States had constructed or initiated numerous large multiple-purpose projects in other sections of the nation on a rate basis which abandoned the competitive plan previously applied, in favor of a power rate fixed by the amount needed to amortize such part of the investment as was determined to be allocable to power, plus costs of operation, maintenance, replacements, etc. This policy was later generalized by statute as to all reclamation projects. Several early determinations of allocations, particularly for navigation and flood control, appeared to be on a relatively liberal basis, resulting in power rates very unfavorable for the Southwest, as regards regional competition for heavy industry and electro-metallurgical plants. Consequently, the Boulder Canyon Project power contractors found themselves obligated to pay, until 1945, a rate considered excessive whether measured by the requirements of the Act or externally, by comparison with the newer projects.

Accordingly, a request for a review of the rates was filed in 1937 with the Secretary of the Interior, in line with a right granted when the initial allocation of the project energy was made. Preliminary conferences showed that a change from the competitive to the amortization basis, however, involved unavoidable differences of interest among the seven states of the Colorado Basin. Two years of almost continuous discussion and negotiation followed, with frequent formal conferences, before all such interests were brought into agreement. As the surplus revenues of the original Act would be entirely eliminated on an amortization basis, determination by compromise and agreement was necessary as regards payments to be made in lieu of taxes to Arizona and Nevada, and additional payments to the Colorado River Basin Development Fund, for the benefit of all seven of the states involved.

Numerous water problems had required previous conferences, in efforts to avoid litigation among the states, so that the power problems came naturally before the same conference groups under the auspices of the several governors and most frequently under the chairmanship of the late Hon. Henry H. Blood, then governor of Utah. As the governors were generally unable to give the time personally for the more frequent conferences, and the difficulty of changing personnel and lack of continuity of program between sessions became apparent, the now well-known "Committee of 14 and 16" began to function. It consists of two representatives of each of the seven basin states, appointed by each governor or by state boards appointed by him, as its only authority is derived from the various governors, being without legislative authorization. To this Committee of 14 are added, for consideration of power problems, two representatives of the California power contractors (Los Angeles and the Metropolitan Water District) which has the further effect of equalizing the basin representation. The committee has operated under the very able chairmanship of Clifford H. Stone of Colorado, to whom in a large measure is due the credit for reaching an harmonious agreement among the states in 1939 on the question of allocation of Boulder Canyon Project power revenues. The committee recommendation that Arizona and Nevada should each receive $300,000 annually, with $500,000 annually going to the Colorado River Basin Development Fund, was adopted by Congress in the Boulder Canyon Project Adjustment Act, approved July 19, 1940.

In view of the immediate benefits to be derived by the three Lower Basin states, a further agreement was made that expenditures from the Development Fund should be limited to the Upper Basin states until 1955. The first three annual payments were applied to the completion of surveys and studies for a "Comprehensive Plan of Basin Development," long under way by the Bureau of Reclamation. Division of the fund among the Upper Basin states was required to be on an "equitable" basis (whatever that might mean), and not until October, 1941, was agreement finally reached to interpret this as meaning "equally." This is an example of the committee's help in avoidance of litigation over the allocation of the project revenues, although protracted discussions were required at times. Without such a forum to give an opportunity for thorough debate of the controversial problems, and final agreement by compromise before the questions could become political issues at Washington, it seems that passage of the Adjustment Act would have been impossible. Even then, however, difficulties were not ended, as this Act required definite repayment contracts to be entered into by June 1, 1941, covering 90% of the firm power or otherwise the Act would be void. Not until May 29, 1941, were the contracts finally signed, making the Adjustment Act effective.

Hearings on these contracts had been convened at Los Angeles on August 12, 1940, continuing until December 6, before a special board of three representing the Secretary of the Interior. The resulting record includes about 2,000 pages of transcript of oral evidence and argument, besides 91 exhibits, many of these being lengthy reports. Chief questions at issue involved details of the agency (generating) contracts; generating charges; time and method for future readjustments of rates, to insure full repayment to the government; O & M cost

estimates; allowances for replacement and depreciation; estimates of firm and secondary energy, available and also marketable; and of greatest interest, the new energy rate. Only the latter point appears to be of importance to the purpose of this paper, as its determination finally achieved the definite cost and revenue allocation that had been lacking under the terms of the original Act. Slight readjustments are still provided for, as of June 1, 1947, and at 5-yr intervals, to correct for excesses or deficiencies in estimated revenues; and annually, to allow for the effect of variations in actual cost of operation, maintenance, and replacements. The latter adjustments increased the firm power base rate (fixed at 1.163 mills per kw-hr for 1937 to 1942) to 1.172 for 1943 and 1.190 for 1944. A proportionate decrease to 0.34 mill per kw-hr for secondary power in the years 1937 to 1942 was changed to 0.346 for 1943 and 0.357 for 1944. The annual secondary rate is to be computed from future adjusted firm rates as the average of (1) a rate which bears the same ratio to the firm rate as 0.3 to 1.1, and (2) a rate which is 0.8 mill less than the firm rate; or otherwise expressed, as the firm rate times 0.6363636, less 0.4; but not less than 0.2 mill per kw-hr.

With similar infinitely detailed and legalistic exactitude, every possible or imaginable contingency of the future as affecting the power rate structure has been provided for at painful length in the several contracts and general regulations, now in force and effect. Revising the original power allocations to allow for the increased power production with the 25-ft higher (575 ft) dam, which change had caused the first allocations to total in excess of 100%, the present power contracts result in the following firm energy allocation:

Allocation	Percentages
Metropolitan Water District of Southern California (for aqueduct pumping only)	35.2517
State of Nevada (for use in Nevada only)	17.6259
State of Arizona (for use in Arizona only; no contract as yet)	17.6259
City of Los Angeles, Calif.	17.5554
Southern California Edison Company	7.0503
City of Glendale, Calif.	1.8475
City of Pasadena, Calif.	1.5847
California Electric Power Company	0.8813
City of Burbank, Calif.	0.5773
Total	100.0000

The power blocks for the two states are equally underwritten by the City of Los Angeles and Southern California Edison Company, with drawback privileges as formerly. Not to exceed 20,000-kw peak demand is reserved for the United States, to be charged equally against the Los Angeles and Edison Company allocations for distribution in Boulder City or for other government camps and uses. This is in addition to power for dam operation, which is to be furnished without charge and included with station losses.

Conforming to the exact terms of the Adjustment Act, the general regulations and the foregoing contracts, a detailed financial operations study now

indicates estimated project revenues over the 50-yr period to May 31, 1987, of $256,303,360, which are sufficient to repay the reimbursable investment in the dam and power-plant building (estimated for rate determinations at $82,878,131 as of June 1, 1941, including 3% interest during construction, or about 82.5 million dollars present value as of June 1, 1937, including some allowance for deferred construction), within fifty years amortized at a 3% interest rate. In addition, all estimated operation, maintenance, and replacement costs are covered, and Arizona and Nevada will each receive $15,000,000 during the fifty years, with $25,000,000 going to the Colorado River Basin Development Fund.

These allocations are compared in Table 2, on a present-value basis, as in Table 1, with the results under the original Act for various firm power rates after 1945, and with the various payments capitalized at 3%. Table 2 shows that, under the terms of the Adjustment Act, Arizona and Nevada will each receive 4.2 times as much revenue in lieu of taxes from the Boulder Canyon Project as they were assured of receiving by the original Act, and that only in case the average revised firm power rate after 1945 had exceeded 1.52 mills per kw-hr would the states have received as much revenue as they are now assured of getting. This is equivalent, on a present-value basis, to 18% of the investment in the dam and power-plant building. Likewise, the Colorado River Basin Development Fund will receive annual payments with a present value

TABLE 2.—ALLOCATION OF ESTIMATED REVENUES AND COSTS, AS PRESENT VALUES, AT 3%, AS OF MAY 31, 1937

Firm power rate (mills per kw-hr)	DAM AND POWER-PLANT BUILDING[a]		REPAYMENT[b]		ARIZONA AND NEVADA		COLORADO FUND[c]	
	Million dollars	Percentage of investment	Million dollars	Percentage of allocation	Million dollars	Percentage of investment	Million dollars	Percentage of investment
(a) POWER RATE, ETC., OF THE ADJUSTMENT ACT								
1.163	82.5	100	....[d]	....[d]	14.9	18	12.4	15
(b) VARIED POWER RATES AFTER 1945[e] FOR COMPARISON WITH ADJUSTMENT ACT								
1.00	97.0	118	5.7	23	3.5	4	0.0	0
1.08	103.6	126	5.7	23	3.5	4	0.0	0
1.163	110.3	134	5.7	23	3.5	4	0.0	0
1.30	110.3	134	12.8	51	7.7	9	0.0	0
1.40	110.3	134	18.6	74	11.3	14	0.0	0
1.555	110.3	134	26.3	105	15.8	19	0.0	0
1.63	110.3	134	30.2	121	18.1	22	0.0	0

[a] Repaid within fifty years. [b] Flood-control allocation repayment. [c] Colorado River Basin Development Fund. [d] Deferred beyond the 50-yr amortization period. [e] Present values of costs and payments required by the original Act but capitalized at 3% interest rate (as allowed by the Adjustment Act).

of 15% of the project investment, instead of the probability (almost certainty) of obtaining nothing under the original Act. A general understanding of these comparisons, at least on an approximate qualitative basis, explains the high degree of agreement that was finally attained in compromising the interests of the various states of the Colorado River Basin.

By the Adjustment Act, the government obtained certainty as to the amortization of its investment, less the flood-control allocation, within a 50-yr period, at a 3% interest rate, pronounced reasonable under the circumstances. Repayment of the flood-control allocation is definitely deferred beyond the 50-yr period; but the 1945 revision of power rates, under the original Act, would almost certainly have accomplished the same effect, at least as to 77% of the allocation.

On the other hand, assuming for purposes of illustration that the new firm power rate of 1.163 mills per kw-hr is fair and reasonable on a competitive market basis (as seems indicated by its acceptance by all of the power contractors), it may be considered that, on a 3% present-value basis, the government surrendered the possibility of a profit of about $27,800,000 (the difference between 4% and 3% interest charges) besides deferring for fifty years the dependable repayment of $5,700,000 toward the flood-control allocation. The recipients of this relinquished profit vary for different assumptions of future competitive rates, but, on the basis of the present rate as stated, the "split-up" would appear to be as follows:

Arizona (additional revenue)	$ 5,700,000
Nevada (additional revenue)	5,700,000
Colorado River Basin Development Fund (additional revenue)	12,400,000
Power contractors (reduced payments, 1937 to 1945)	9,700,000
Total	$33,500,000

The savings to the power contractors will be divided among power consumers in Arizona, Nevada, Utah (slightly), and Southern California, the chief beneficiary; but this is considerably less than the 100-million-dollar gain which the power contractors were often asserted to be obtaining, during the years of consideration of the Adjustment Act. With the Colorado River Basin Development Fund probably to be divided about equally among the four Upper Basin states until 1955, the indicated advantage of the Adjustment Act to each Upper Basin state is computed at $2,600,000, and to each of the states of Arizona and Nevada, $6,700,000 in addition to $1,750,000 apiece to the latter two under the terms of the original Act (all amounts as of June 1, 1937, discounted at 3%). After 1955 it may benefit Nevada and Arizona projects also. California is most unlikely to share in the fund then, because of the earlier completion of all projects having an available water supply. It is not implied that the 1945 revised firm power rate, by the original Act on the basis of competitive market conditions, would have been exactly 1.163 mills per kw-hr or even approximately this amount; but a similar comparison of individual benefits can be derived from Table 2 for any other revised rate that may be assumed. The illustration given, however, shows at least that the division of the benefits allocated by the Adjustment Act was not merely or solely in proportion to the Congressional voting strength of the various states, for example. Other factors also received consideration, such as anticipated receipts under the terms of the original Act and competitive power market conditions, regional as well as local.

The government, of course, ends by owning the entire project including replacement and depreciation funds for items subject to loss with use, and also retains final discretion as to the expenditure of the Colorado River Basin Development Fund. The foregoing "split-up" of this fund within the Upper and Lower basins, prior to and after 1955, is simply in accordance with the recommendation of local interested officials, which Congress may or may not follow exactly.

In refreshing contrast, from an engineering point of view, to the foregoing allocations, which are so largely dependent on legal and accounting, as well as political, considerations, is a novel rate item in the November, 1941, issue of *Intake* (Los Angeles Water and Power Department publication). New concepts in physics establish a definite relationship between energy and matter, making it possible to assign a weight to electrical energy and compare it with other commodities on the basis of cost per pound. At present Los Angeles retail prices, electricity costs about $20,000,000 per lb or two thirds the price of radium. As the power production from the Boulder Canyon Project in 1943 was about six "pounds" of electrical energy, on this futuristic basis, the adjusted falling water charge is equivalent to $900,000 per lb, or about the market valuation for fine, large cut diamonds. Although this fact certainly was not given much, if any, consideration in the project's cost allocation and power rate determination, to an engineer fully appreciative of the Boulder Canyon Project, it does not seem too far-fetched to value the output of the project as highly as precious jewels.

Summary

Revised power contracts for the Boulder Canyon Project (Arizona-Nevada), dated May 29, 1941, provided for the amortization of the cost, at 3% interest rate, by terms of the Adjustment Act of July 19, 1940. A power (falling water) charge of 1.163 mills per kw-hr was reduced from the 1930 rate of 1.63 mills per kw-hr, as the original Act's competitive basis for power rate determination was abandoned. Depression conditions had affected adversely the competitive rate factors (price of oil and natural gas, cost of steam plants, etc.) so that the rate revision required by the original Act, in 1945, would probably have resulted in approximately the same or perhaps a greater rate reduction.

This loss of revenues would have decreased or even eliminated the project payments to Arizona and Nevada, which are considered to be in lieu of taxes, and also would have ended the chance of payments to the Colorado River Basin Development Fund which were not available in any case until about 1985. Even the repayment of the government's investment in the dam and power-plant building might not have been completed within fifty years as planned. Therefore, by agreement among the United States, the power contractors, and the seven Colorado River Basin states, as represented by their "Committee of 14 and 16," the interest rate was reduced from 4% to 3%, repayment of $25,000,000 flood-control allocation was deferred beyond the 50-yr period, but amortization of balance of investment was made certain within this period, and definite annual payments to states were provided. These include $300,000 apiece to Arizona and Nevada, and $500,000 to the Colorado River Basin Development Fund, limited to use in the Upper Basin only, until 1955.

In this paper, project revenues made available by the present revised contracts and the Adjustment Act, derived from power sales and also water storage service charges, are tabulated by allocations to repayment of investment, payments in lieu of taxes, contributions to the Colorado River Basin Development Fund, and flood-control allocation repayments. For comparison, similar allo-

TABLE 3.—ALLOCATION OF BOULDER CANYON PROJECT COSTS EXPRESSED AS PERCENTAGES OF INVESTMENT FOR DAM AND POWER-PLANT BUILDING,[a] ON THE BASIS OF PRESENT VALUES OF JUNE 1, 1937, AT INTEREST RATES OF 3% AND 4%

Power charges, in mills per kw-hr	Investment in Dam and Power Plant[a]		Arizona and Nevada, in Lieu of Taxes		Development Fund[b]	Flood-Control Allocation of $25,000,000			
						Normal[c]		Alternate[d]	
	3%	4%	3%	4%	3%	3%	4%	3%	4%
(*a*) Adjustment Act and Revised Contracts									
1.163	96%[e]	....	18%	....	15%	....[f]	....[f]	....[f]	....[f]
(*b*) Original Act and Contracts									
1.63[g]	130%[e]	97%[h]	22%	16.4%	0	36%	27%	121%	100%
1.163[g]	130%[e]	97%[h]	4%	3.7%	0	7%	6%	23%	22%
1.0[g]	114%[e]	85%[h]	4%	3.7%	0	7%	6%	23%	22%

[a] Less $25,000,000 flood-control allocation. [b] Colorado River Basin Development Fund. [c] In terms of the remainder of reimbursable project cost. [d] Expressed as a percentage of the flood-control allocation of $25,000,000. [e] Water storage charge, 4%. [f] Deferred until after 1987, or the end of the 50-yr amortization period. [g] Power rate, as it might have been revised after 1945 by the terms of the original Act. [h] Water storage charge, 3%.

cations under the terms of the original Act and contracts are presented in Table 3, assuming varied power rates after the 1945 adjustment date, and computing present values at both the adjusted 3% interest rate and at the 4% rate of the original Act. The change in interest rate was made on a showing that the interest cost to the U. S. Treasury for long-term loans during the project construction period did not exceed 2.75%.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

FAILURE OF THE TACOMA NARROWS BRIDGE

REPORT OF THE SPECIAL COMMITTEE OF THE BOARD OF DIRECTION

Committee Authorization and Meetings

Authorization.—At the Annual Convention of the Society, held at San Diego, Calif., in July, 1941, the Board of Direction authorized the appointment of a committee to review the official reports in regard to the failure of the Tacoma Narrows Bridge in the State of Washington, and to submit a report. At the next Fall Meeting (Chicago, Ill., October 14, 1941) the Board supplemented these instructions by passing the following resolution:

> "That the Committee be given the opportunity to make its own recommendations and extend its investigations, if deemed wise, and to make its own comments."

Meetings.—The committee held two meetings, one on October 15, 1941, during the Chicago Meeting, and the other in New York, N. Y., on January 20, 1942, at the time of the Annual Meeting of the Society. Each of these meetings was attended by all members of the committee. O. H. Ammann and the late L. S. Moisseiff, Members, Am. Soc. C. E., were also present by invitation at the latter meeting.

Attendance of Chairman at Meeting of Bridge Engineers in New York.—The committee also participated, through its chairman, at a meeting in New York City on Thursday, December 13, 1941, at the office of the American Institute of Steel Construction, which was attended by the following: Shortridge Hardesty (represented by J. M. Garrelts), Charles M. Spofford, Raymond Archibald, Charles E. Andrew, O. H. Ammann, L. S. Moisseiff, Jonathan Jones, C. F. Goodrich, and F. H. Frankland, Members, Am. Soc. C. E., Blair Birdsall, Assoc. M. Am. Soc. C. E., with Messrs. E. F. Kelly and C. C. Sunderland. This group made the following recommendations to the U. S. Bureau of Public Roads,

> "Whereas it appears that the investigation of the safety of suspension bridges and their adequate design under aerodynamic forces is of such national importance that it should be initiated and carried out by the proper Federal authority; the undersigned respectfully request that such a program of investigation be initiated and carried out by the Public Roads Administration."

Note.—Written comments are invited for immediate publication in *Proceedings;* to insure publication the last discussion should be submitted by **May 1, 1944.**

This recommendation led to the appointment of a committee of twenty five by Thomas H. Macdonald, U. S. Commissioner of Public Roads, of whom all but three were corporate members of the Society.

Through the courtesy of the proper authorities, the Society's committee secured copies of the reports listed hereafter, all of which were reviewed by the committee; and their conclusions are discussed in detail in the various appendixes to this report.

Reports

List of Reports Reviewed by Committee.—Five formal reports were examined by the Society's committee:

(*a*) "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., dated March 28, 1941, by a Board of Engineers consisting of Othmar H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Members, Am. Soc. C. E. (see Appendix I).

(*b*) Report in three parts of the Board of Investigation, Tacoma Narrows Bridge, appointed by James A. Davis, Acting Director of Highways for the State of Washington, Part 1, dated February 17, 1941, Supplemental Report of March 4, 1941, and Part 2, dated June 26, 1941. The Board consisted of the following: L. J. Sverdrup (chairman) and Russell G. Cone, Members, Am. Soc. C. E., with Francis Donaldson (see Appendix II).

(*c*) "The Failure of the Suspension Bridge over Tacoma Narrows, a Report of the Board of Consulting Engineers to the Narrows Bridge Loss Committee (Paul Carew, Chairman) on the Extent of Damage Produced by the Failure, Dated June 2, 1941," the Board consisting of Clifford E. Paine (chairman), Hardy Cross, Shortridge Hardesty, Holton D. Robinson, and Wilbur M. Wilson, Members, Am. Soc. C. E. (see Appendix III).

(*d*) Another report by the Board named under (*c*) entitled "Proposed Reconstruction of the Suspension Bridge Over Tacoma Narrows," dated June 2, 1941.

(*e*) A report entitled "Laboratory Studies on the Tacoma Narrows Bridge," addressed to Burwell Bantz, Director of Highways, Olympia, Wash., and conducted under the joint auspices of the Public Roads Administration for the Washington State Highway Department and the University of Washington by F. B. Farquharson, M. Am. Soc. C. E., Professor of Civil Engineering of the University of Washington, Seattle, Wash., and a member of this committee. The tests presented in Report (*e*) were made in the laboratories of the University of Washington (see Appendix IV).

In addition, the committee secured copies of researches made in the Engineering Societies Library covering the suspension bridge between Covington, Ky., and Cincinnati, Ohio. A Bibliography on the subject is presented as Appendix V.

General Features of the Bridge Project

Location.—The Tacoma Narrows Bridge was built across Puget Sound near the City of Tacoma, Wash., to connect the Olympic Peninsula with local high-

FIG. 1.—LOCATION MAP OF TACOMA NARROWS BRIDGE

ways. The Sound is 4,600 ft wide at its narrowest point and the waterway is deep, with swift tidal currents. The bridge was intended to replace the ferries which previously had been the only means of direct communication in this vicinity between the mainland and the peninsula, although there are highways around the southern end of the Sound. The distance from Tacoma to the Bremerton Navy Yard by highway is 96 miles. The bridge offered a saving of 59 miles in this instance. The bridge location and character of terrain are shown clearly in Figs. 1 (from Report (*a*)) and 2 (from Report (*e*)).

History.—The bridge was constructed during the years 1938–1940 by the Washington Toll Bridge Authority, established by the State Legislature for the purpose of constructing and operating toll bridges, and consisting of the Hon. Clarence D. Martin, Governor; Cliff Yelle, State Auditor; Lacey V. Murrow, Assoc. M. Am. Soc. C. E., Director of Highways; Olaf L. Olsen, Director of Finance and Budget; and P. H. Winston, Secretary of the Authority.

According to the Carew Report the contract for construction was awarded on November 23, 1938, as a joint venture, to the Pacific Bridge Company, the General Construction Company, and the Columbia Construction Company, the low bidder, for a lump sum of $5,949,730.40. The changes and additions during construction increased the construction cost to $5,998,230.68. The cost of administration, engineering, interest, insurance, taxes, and other costs not included in the contract, but incurred prior to September 30, 1940, amounted to $471,539.32, bringing the total cost of the bridge to $6,469,770.

The steel superstructure was manufactured and erected by the Bethlehem Steel Company, the wire and suspender ropes being furnished by the John A. Roebling's Sons Company. The bridge was opened to traffic on July 1, 1940, and failed on November 7, 1940.

Financing.—Officials of the State of Washington were of the opinion that it would not be financially feasible to construct the bridge without federal assistance; and even with such assistance it was considered necessary to keep the cost of construction as low as possible. However, judging from the amount of traffic using the bridge during its short life, the estimates of traffic and revenue may have erred on the low side. An application for federal assistance was made and on June 27, 1938, the Public Works Administration (PWA) made a grant of $2,700,000 and a loan of $3,300,000 was made by the Reconstruction Finance Corporation. In October, 1938, after bids for the construction, amounting to $5,949,730.40 had been received, the grant and loan were increased to $2,880,000 and $3,520,000, respectively, making a total of $6,400,000. The loan by the Reconstruction Finance Corporation was repaid by a refunding operation shortly after the bridge was opened to traffic.

Insurance.—The bridge was insured against direct loss for $5,200,000; it was also insured against loss of use and occupancy to the amount of $240,000. Insurance to the amount of $4,000,000, plus occupancy insurance for $240,000, was paid by the insurance companies.

Engineering.—The engineering work was the responsibility of the following: Lacey V. Murrow, Assoc. M. Am. Soc. C. E., chief engineer; Charles E. Andrew, M. Am. Soc. C. E., principal consulting engineer; Luther E. Gregory, M. Am. Soc. C. E., consulting engineer; R. H. Thomson, Hon. M. Am. Soc. C. E., consulting engineer; R. B. McMinn, consulting engineer; Moran, Proctor,

Freeman and Mueser, consultants on foundations; Leon S. Moisseiff, M. Am. Soc. C. E., consultant on superstructure; and C. H. Eldridge, M. Am. Soc. C. E., bridge engineer.

The contract drawings were signed by Mr. Murrow as state highway engineer and Mr. Eldridge as bridge engineer. They also carry the approval of Moran, Proctor, Freeman and Mueser as consultants on the substructure and Mr. Moisseiff as consultant on the superstructure.

In addition to the foregoing engineering staff, the plans were given an independent but limited review by T. L. Condron, M. Am. Soc. C. E., for the

FIG. 2.—VIEW OF BRIDGE SITE AND COMPLETED BRIDGE, FACING NORTH

Reconstruction Finance Corporation, which purchased the bonds (see Appendix IV of Report (*a*)). It is interesting to observe that, although Mr. Condron made no attempt to go into details of the design, he commented unfavorably on the relative narrowness of the structure but deferred to the wider experience of the other consultants.

Dimensions and Composition.—This suspension bridge had a center span of 2,800 ft and two side spans of 1,100 ft each. It had a roadway 26 ft wide and two sidewalks 4 ft 9 in., with a total width, center to center of stiffening girders, of 39 ft. There were also plate-girder approach spans which with the anchorages made the over-all length 5,939 ft.

The main towers were 420 ft high, and were composed of two shafts, each uniform in section, connected by horizontal struts and fixed at the bases. The shafts were spaced 39 ft centers at the top and 50 ft at the base. Each cable consisted of nineteen strands of 332 No. 6 cold-drawn galvanized wire. The diameter of each cable under wrapping was $17\frac{1}{8}$ in. The net area of wire was 191.296 sq in. The cable had a sag under dead load of 232 ft, giving a sag ratio of 1 : 12. Suspenders were placed 50 ft apart along the cable and each consisted of four parts of $1\frac{1}{4}$-in. ropes.

Cables were stiffened by one silicon steel plate girder on each side consisting of a 96-in. by $\frac{1}{2}$-in. web, two 8-in. by 6-in. by $\frac{1}{2}$-in. flange angles top and bottom,

and one 20-in. by $\frac{1}{2}$-in. cover plate on each flange. The web was stiffened longitudinally by three Z-bars each 5 in. by $3\frac{1}{4}$ in. by $\frac{5}{16}$ in. and one angle 3 in. by $3\frac{1}{2}$ in. by $\frac{3}{8}$ in. These girders were exceptionally shallow and consequently had little vertical rigidity.

The floor beams were spaced at 25-ft centers. The stringers were 21-in. WF 59-lb beams. Floor beams consisted of 52-in. by $\frac{3}{8}$-in. web and two flange angles at top and at bottom, each 6 in. by 4 in. by $\frac{11}{16}$ in. Supported directly on the stringers was the roadway which was a $5\frac{1}{4}$-in. reinforced concrete slab. Each sidewalk had a $2\frac{1}{2}$-in. reinforced concrete slab.

The bridge was very narrow as compared with other bridges of even approximately its span; hence the torsional resistance to unsymmetrical loads or gusts was unusually small. This lack was compensated for to some extent by the smaller sag and increased cable loading with resultant increase in cable stiffness.

Behavior of Bridge Prior to Failure

Durkee Report.—Quoting from the Carmody Report (*a*) (pages 28 and 29):

"The following is quoted from a letter of L. R. Durkee, project engineer, dated November 4, 1940:

'(1) Motions of considerable magnitude, having amplitudes as high as 48″ with frequencies of 16 per minute, have been observed with wind velocities as low as three or four miles per hour.

'(2) Motions of varying degrees of violence have been noted in winds up to 48 miles per hour. The violence of motion is not necessarily proportional to the velocity of wind.

'(3) The bridge has remained motionless at times in wind velocities varying from zero to 35 miles per hour.

'(4) There appears to be no difference in the motion whether the wind is steady or gusty.

'(5) Traffic loads appear to have no measurable effect on the motion.

'(6) Altogether, seven different motions have been definitely identified on the main span of the bridge, and likewise duplicated on the model [tested at the University of Washington]. These different wave actions consist of motions from the simplest, that of no nodes, to the most complex, that of seven nodes. The seven-node motion has been observed but once and had a frequency of 30 cycles per minute.

'(7) Amplitudes as high as 60″ (over all motion plus to minus), have been observed.'

"Excerpts of other reports from Mr. Durkee follow:

"Sept. 18, 1940. 'Movement appears to be caused by wind or air currents and exist from wind velocities of one or two miles per hour up to the highest observed velocities of thirty-five miles an hour. These movements are vertical and have approached a maximum magnitude of about fifty inches and a maximum rate of twenty-seven complete oscillations per min. The most common type appears to be that which is designated No. 3 * * *. [This type has two nodes in the center span.] These movements do not occur continuously but for irregular intervals of time from a few moments up to six or eight hours. While no definite data are available, it is estimated that of the total time, these movements are probably occurring 10 to 15% of the time.

"Traffic has and is using the bridge continuously without other than the unusual sensation experienced. Close observations have been kept to observe any structural effect from these movements but none has been noted.' * * *

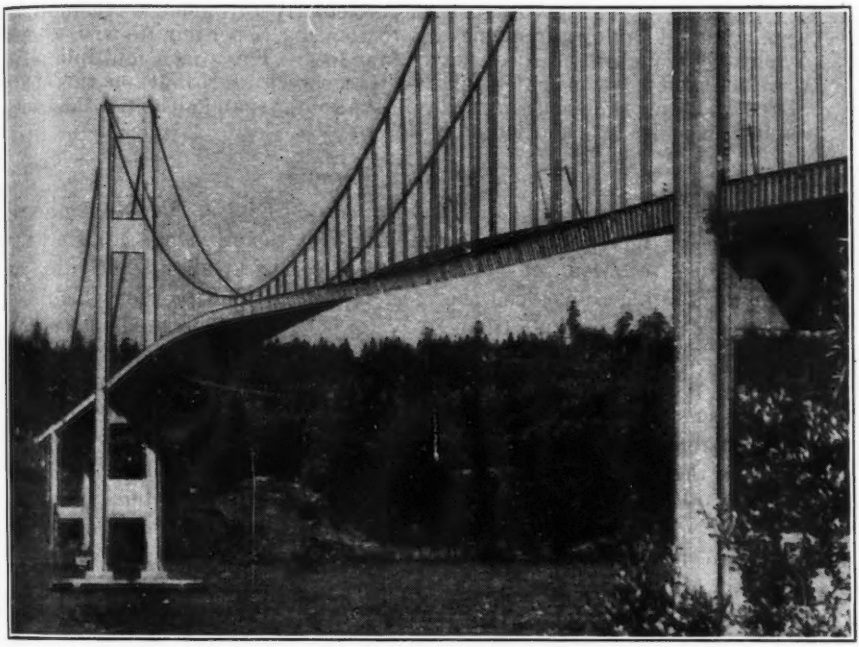


FIG 3.—VIEW OF THE BRIDGE ONE HOUR PRIOR TO COLLAPSE

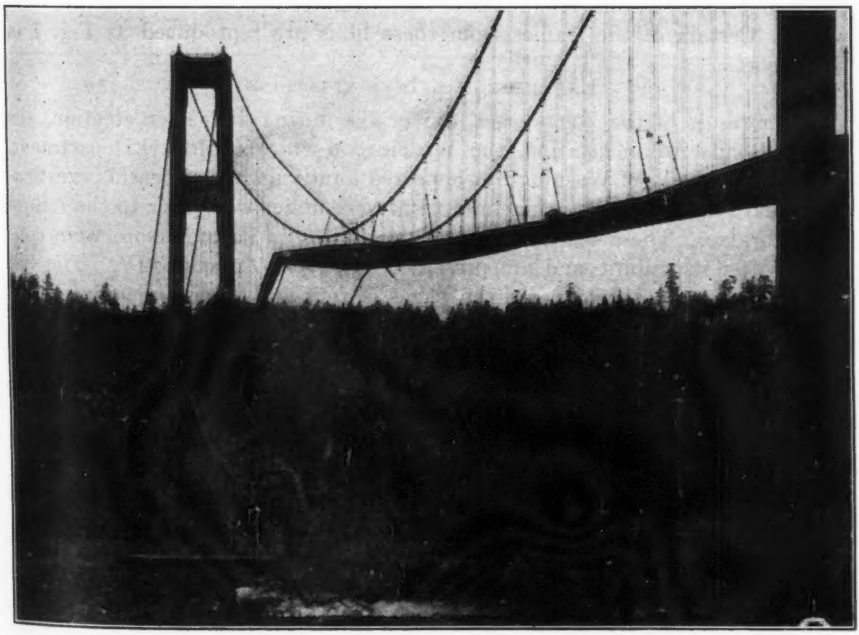


FIG. 4.—VIEW OF BRIDGE DURING FAILURE

"Sept. 19, 1940. 'On August 26, particularly turbulent wave motions were observed having a frequency approximating 27 per minute with a total amplitude, trough to crest, of nearly four feet. This was a multiple node wave movement somewhat similar to that shown as No. 10 on the chart (Type 8A) [this type has four nodes in the center span] although the exact number of nodes was not definitely determined.' "

Description of Failure

Date and Wind Velocity and Vertical Movement of Bridge.—On November 7, 1940, the wind, which had been blowing with increasing velocity during the night, reached a recorded velocity of 42 miles per hr at about 9:00 a.m., at which time the bridge developed the most violent vertical motion which had been recorded, moving at the rate of 36 cycles per min. This mode of motion involved nine half waves with eight nodal points in the main span.

Torsional Movement of Bridge.—Suddenly at approximately 10 a.m. of that day the center span developed a torsional movement with a node at midspan, the angle of rotation at the quarter point being at times nearly 45° and the amplitude of the vertical motion of the girder approximately 28 ft (see Fig. 3).

Method of Failure.—As a result of torsional movement the center span of the bridge finally failed, portions of the center span of the suspended structure dropping into the stream (Fig. 4) and as a result the side spans sagged and the towers were bent shoreward a maximum deflection of about 24 ft (Figs. 5 and 6). (Figs. 3, 4, 5, and 6 are from the Carmody Report (*a*).) Fortunately the failure was gradual and no lives were lost. Motion pictures made at the time by Professor Farquharson, of the committee, showed the motions and failure of the bridge in a striking manner and were produced in newsreels all over the world. Certain of the frames from these films are reproduced as Fig. 7 in this report.

Experimental Investigations

As a result of the behavior of the bridge during and after erection, the Public Roads Administration, the Washington State Highway Department, and the University of Washington sponsored jointly an experimental investigation upon a model of the bridge and tests were under way prior to the failure of the bridge. These tests, conducted by Professor Farquharson, were continued after its failure, and are presented briefly in Appendix IV. The full report (*e*) contains 101 typewritten pages of text and three appendixes, consisting respectively of: (A) Thirty-two photographs of models as mounted in a wind tunnel; (B) twelve diagrams showing the effect of various conditions in the model tests; and (C) forty-seven pages of text and four pages of illustrations relating principally to wind failures and reconstruction of the following suspension bridges: Montrose, Scotland, on October 11, 1838; Brighton Chain Pier on November 30, 1836; and Menai Strait on January 7, 1839.

Report (*e*) also contains a copy of an article by John Scott Russell, vice-president of the Society of Arts in Scotland, presented to the society on January 16, 1839, entitled "On the Vibrations of Suspension Bridges and Other Structures; and the Means of Preventing Injury from this Cause" (5);[1] also an article by W. Reid, Royal Engineers, concerning a failure that occurred on November 30, 1836. The latter appears as Appendix VI of this report.

[1] Numerals in parentheses, thus: (5), refer to corresponding items in the Bibliography (see Appendix V).

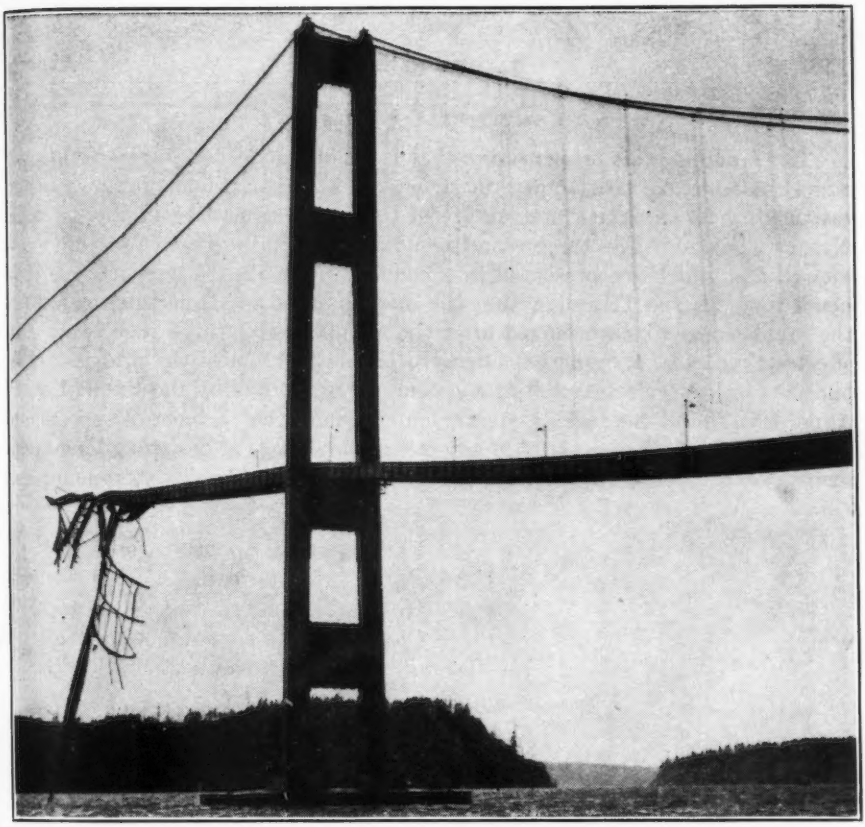


FIG. 5.—CONDITIONS NEAR THE WEST TOWER AFTER FAILURE

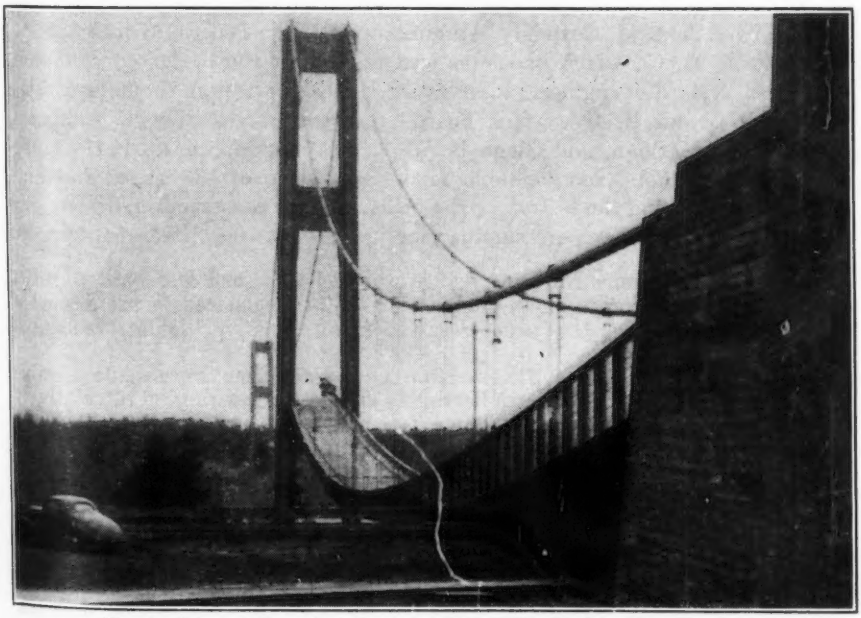


FIG. 6.—VIEW OF EAST SIDE SPAN AFTER THE CENTER SPAN HAD DROPPED

Committee Conclusions

The failure of this bridge shows that the methods of design computations, which had proved satisfactory on structures of other proportions, were not satisfactory for designing a structure of the proportions used in the Tacoma Narrows Bridge. The three reports on the bridge failure which we have reviewed and which are presented in a condensed form in this report (see Appendixes I, II, and III) show that the methods used in attempting to reduce the oscillations, which occurred after the completion of the bridge, were not effective, although they perhaps delayed the destruction of the bridge. It is hoped that further research by the committee appointed by the Public Roads Administration of the federal government to study the problem of suspension bridge design will be successful in advancing the science of designing long-span suspension bridges so that they will safely resist both static and dynamic forces.

James H. Cissel
Frederick B. Farquharson
Frank M. Masters
Homer R. Seely
Charles M. Spofford, *Chairman*
Committee on the Tacoma Narrows Bridge

APPENDIX I

Review of the Carmody Report (*a*)

The Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., which had advanced money for the bridge construction, appointed a board of engineers to investigate and report upon the failure of the Tacoma Narrows Bridge. This board consisted of Othmar H. Ammann, Theodor von Kármán, and Glenn B. Woodruff. On March 28, 1941, it presented a voluminous and valuable report consisting of 138 pages and nine appendixes, covering the history of the bridge and its construction, design, and failure. The conclusions are summarized in the following fifteen paragraphs:

"1. The Tacoma Narrows Bridge was well designed and built to resist safely all static forces, including wind, usually considered in the design of similar structures. Its failure resulted from excessive oscillations caused by wind action.

"2. The excessive vertical and torsional oscillations were made possible by the extraordinary degree of flexibility of the structure and of its relatively small capacity to absorb dynamic forces. It was not realized that the aerodynamic forces which had proven disastrous in the past to much lighter and shorter flexible suspension bridges would affect a structure of such magnitude as the Tacoma Narrows Bridge, although its flexibility was greatly in excess of that of any other long span suspension bridge.

"3. The vertical oscillations of the Tacoma Narrows Bridge were probably induced by the turbulent character of wind action. Their amplitudes may have been influenced by the aerodynamic characteristics of the suspended structure. There is, however, no convincing evidence that the

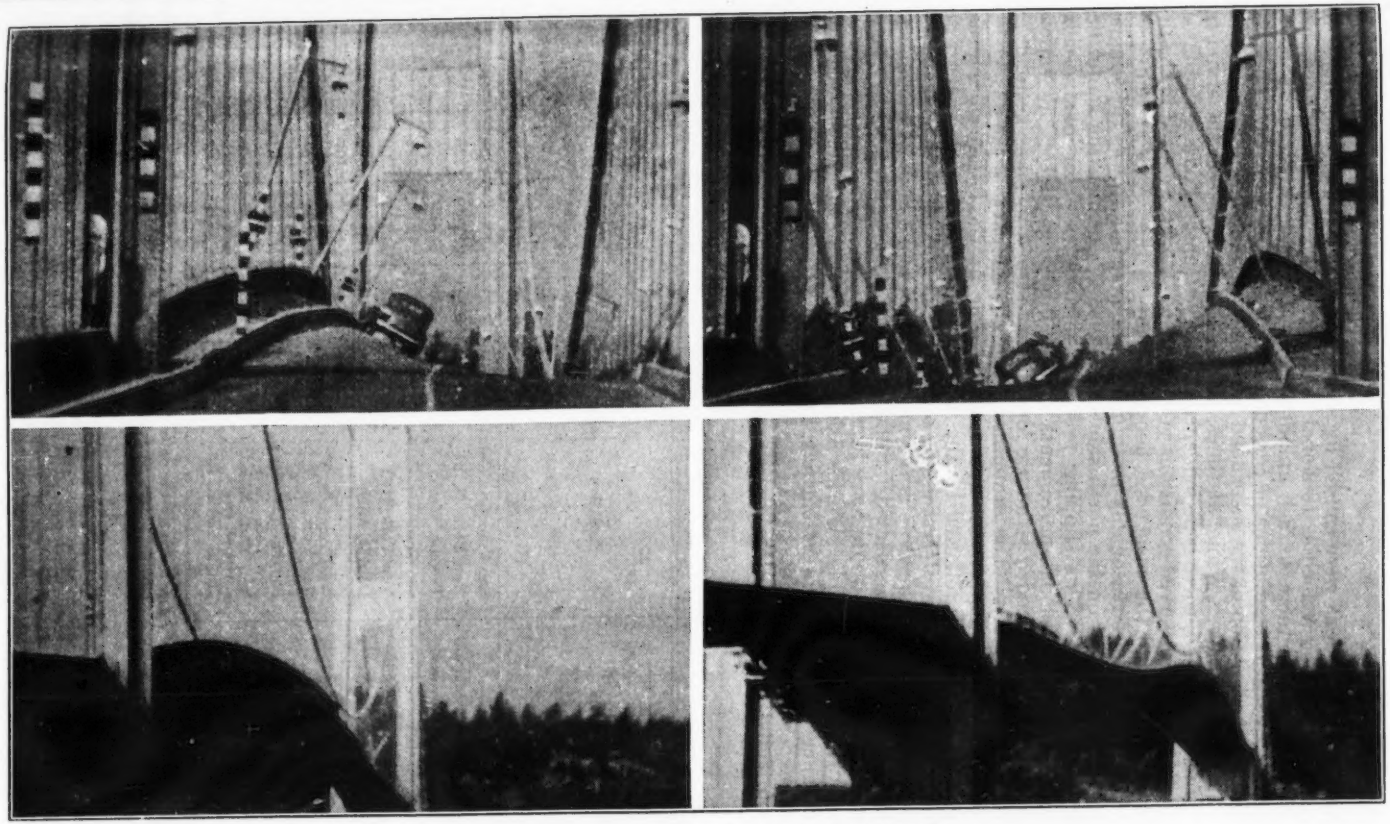


FIG. 7.—OBSERVATIONS IN THE FIELD PRIOR TO COLLAPSE

vertical oscillations were caused by so-called aerodynamic instability. At the higher wind velocities torsional oscillations, when once induced, had the tendency to increase their amplitudes.

"4. Vertical oscillations of considerable amplitudes were first observed during the erection of the suspended floor and continued, at intervals, until the day of failure. While, at times, the resulting stresses in the stiffening girders were high, there is no evidence that any structural damage resulted. Under certain observed conditions very high stresses were caused in the ties which connected the suspended floor structure to the cables at mid-span.

"5. It appears reasonably certain that the first failure was the slipping of the cable band on the north side of the bridge to which the center ties were connected. This slipping probably initiated the torsional oscillations. These torsional movements caused breaking stresses at various points of the suspended structure and further structural damage followed almost immediately. The dropping of the greater part of the suspended structure of the center span was made possible by the failure of the suspenders. This was followed by the sudden sagging of the side spans with resulting bending and overstressing of the towers and of the side spans.

"6. The suspension type is the most suitable and the most economical that could have been selected for the Tacoma Narrows Bridge. No more satisfactory location would have been chosen.

"7. Both the Public Works Administration and the Reconstruction Finance Corporation were entirely justified in assuming that, because of the experience and reputation of the consultants employed by the Washington Toll Bridge Authority, there could be no possible question as to the

TABLE 1.—DIMENSIONS OF SUSPENSION BRIDGES

Item	Name	Location	Completed	Feet				
				Center span	Side spans	Cable sag	Width[a] between cables	Depth[b]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Golden Gate	San Francisco, Calif.	1937	4,200	1,125	475	90	25
2	George Washington	New York, N. Y.	1931	3,500	610–650	325	106	29[d]
3	Tacoma Narrows	Tacoma, Wash.	1940	2,800	1,100	232	39	8
4	Transbay	San Francisco, Calif.	1936	2,310	1,160	231	66	30
5	Bronx-Whitestone	New York, N. Y.	1939	2,300	735	200	74	11
6	Ambassador	Detroit, Mich.	1929	1,850	817[e] 973[e]	209	67	22
7	Delaware River	Philadelphia, Pa.	1926	1,750	716.7	197	89	28
8	Bear Mountain	West Point, N. Y.	1924	1,632	....[c]	200	61.3	30
9	Williamsburg	New York, N. Y.	1903	1,600	596[e]	178	67	40
10	Lions Gate	Vancouver, B. C.	1938	1,550	615	150	40	15
11	Mid-Hudson	Poughkeepsie, N. Y.	1930	1,500	750	150	42	20
12	Manhattan	New York, N. Y.	1909	1,470	725	148.5	96	24
13	Triborough	New York, N. Y.	1936	1,380	705	138	98	20
14	St. Johns	Portland, Ore.	1930	1,207	430	121	52	18
15	Mount Hope	Providence, R. I.	1928	1,200	504	120	34	18

[a] Center to center of cables. [b] Depth of stiffening girder. [c] Number of electric railway tracks. [d] These values 31,500 lb per sq ft and there are no electric railway tracks (Col. 15). [e] Unloaded backstays.

adequacy of the design. Both agencies exercised thorough and competent supervision during the construction of the bridge.

"8. There can be no question that the quality of the materials in the structure, and the workmanship, were of a high order.

"9. Certain parts of the towers were severely overstressed and permanently deformed during the failure. While there is no visual evidence of damage to the cables, except at the center of the north cable, it is probable that they were overstressed during the torsional oscillations and as a result of the sagging of the side spans. The main piers were not damaged, except locally, during the failure and can withstand considerably heavier tower reactions than they received from the bridge as it existed. The anchorages were not damaged and are safe for forces not greater than those imposed by the original construction.

"10. The criteria usually considered for rigidity against static forces do not necessarily apply to dynamic forces.

"11. The remedial installations in the bridge represented a rational effort to control the amplitudes of the oscillations. Further installations, including diagonal stay ropes from the top of the towers to the floor were being investigated when the failure occurred, and these would have increased the rigidity. It is doubtful that any measures of this nature would have been sufficient to compensate for the extreme flexibility of the structure.

"12. The evidence as to whether the vertical oscillations of the bridge would have been affected by fairing (streamlining) is inconclusive. There is certain evidence that fairing would have had an unfavorable influence on the torsional stability.

"13. Further experiments and analytical studies are desirable to investigate the action of aerodynamic forces on suspension bridges.

"14. Pending the results of further investigations, there is no doubt that sufficient knowledge and experience exist to permit the safe design of a

BUILT AFTER 1900, WITH SPANS OF 1,200 FT OR MORE

Lb Per Ft of Bridge		Roadways		Sidewalks		No. tracks[c]	Ratios			Cables		Item
Dead load center span	Design live load	No.	Width	No.	Width		Sag to span	Width[a] to span	Depth[b] to span	No.	Diameter (in.)	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	
21,600	4,000	1	60.0	2	10.0	0	1 : 9	1 : 47	1 : 168	2	36⅜	1
39,000[d]	8,000[d]	2[d] 1	28.75[d] 30.5	2	10.00[d]	4[d] 0	1 : 11	1 : 33	1 : 120[d]	4	36	2
5,700	1,000	1	26.0	2	5.0	0	1 : 12	1 : 72	1 : 350	2	17¼	3
18,800	8,000	1 1	58.0 31.0	0	....	2	1 : 10	1 : 35	1 : 77	2	28¾	4
11,000	3,000	2	27.5	2	5.0	0	1 : 11.5	1 : 31	1 : 209	2	22	5
12,400	3,300	1	47.0	1	8.0	0	1 : 8.9	1 : 27.6	1 : 84	2	19¼	6
26,000	6,000	1	57.0	2	10.0	4	1 : 8.9	1 : 19.7	1 : 62.5	2	30	7
11,540	3,160	1	38.0	2	5.0	0	1 : 8.2	1 : 26.6	1 : 54	2	18½	8
17,200	5,700	2	20.0	2	17.0	6	1 : 9	1 : 23.7	1 : 40	4	18⅝	9
4,600	1,230	1	29.0	2	5.0	0	1 : 10.3	1 : 38.8	1 : 104	2	13¼	10
8,800	3,000	1	30.0	2	4.0	0	1 : 10	1 : 35	1 : 75	2	16⅜	11
23,280	8,000	1	35.0	2	13.0	8	1 : 9.9	1 : 15.3	1 : 61	4	21½	12
20,000	4,000	2	43.5	2	5.5	0	1 : 10	1 : 14.1	1 : 68	2	20¾	13
....	....	1	40.0	2	5.0	0	1 : 10	1 : 23.2	1 : 67	2	16⅜	14
5,300	1,500	1	27.0	1	2.5	0	1 : 10	1 : 35	1 : 67	2	11	15

pply to the ultimate double-deck condition. For the present single-deck condition the dead load (Col. 9) is

suspension bridge of any practicable span. The results of further research should furnish knowledge that will permit of more economical design.

"15. This report has been restricted to the Tacoma Narrows Bridge, except that available information from other bridges has been considered."

The report emphasizes the fact that no provision was made in the design for the aerodynamic wind forces which are so important in the design of airplanes. The effect of such forces in producing vibrations on transmission lines is well known and is the subject of analytical treatment in textbooks dealing with

TABLE 2.—ELASTIC PROPERTIES AND PRINCIPAL DEFLECTIONS COMPUTED

Item	Name of bridge[a]	(a) General				(b) Proper-	
		Dimensions, Feet			Weight[b] (lb per lin ft)	No.	Cable sag (ft)
		Center span length	Side span length	Width between cables			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Golden Gate	4,200	1,125	90	21,035	2	475
2	George Washington	3,500	650	106	31,590[d]	4	319.2[d]
3	Tacoma Narrows	2,800	1,100	39	5,700	2	232
4	Transbay	2,310	1,160	66	18,740	2	231
5	Bronx-Whitestone	2,300	735	74	11,000	2	200

[a] For location and other data, see Table 1. [b] Average weight of center span. [c] Area of net section, all cables. be 39,500 lb per linear foot of bridge and the sag will be 325 ft (see Table 1). [d] Built-up plate girders. [f] Center

TABLE 2.—

Item	(d) (*Continued*)	(e) Properties of Towers					
	Moment of inertia, wind truss (in.[2] ft[2])	Feet			Area, Both Tower Legs (Sq In.)		Moment of inertia[i] (in.[2] ft[2])
		Height of towers[g]	Width[h] at:				
			Top	Bottom	Top	Bottom	
	(18)	(19)	(20)	(21)	(22)	(23)	(24)
1	1,236,000	702	24.6	52.6	7,340	14,384	1,500,000
2	481,000[d]	582	37.5	56	5,978	14,624	3,130,000
3	95,000	425	13	19	1,524	2,287	40,500
4	743,000	462.41	15	32	3,712	8,000	290,000
5	410,000	353	12	18	3,300	4,200	60,000

[g] Top of pier to center of cable. [h] Parallel to the bridge axis. [i] Average for both tower legs. [j] Pounds per linear quarter point of the center span. [m] From design wind load at the center of the center span. [n] Maximum de-

mechanical vibrations. Such a phenomenon is known as galloping, and the Tacoma Bridge had received the nickname of "Galloping Gertie." The report also states that the practice of reducing the vertical stiffness of suspension bridges by using shallow trusses or girders, instead of the deep ones used in the great suspension bridges built prior to the last decade, developed apparently without any consideration of the aerodynamic forces in spite of the fact that much attention has been given to the study of aeronautics in recent years. The builders of this bridge, being limited in funds, were anxious to build as inexpensive a bridge as possible in order to build any bridge at all.

Table 1 (see Report (*a*), Table 3, page 70) shows the dimensions of suspension bridges built since 1900 with spans of 1,200 ft or more, and Table 2 (Report (*a*), Table 4, page 77) gives dimensions, structural properties, vertical and lateral deflections, and longitudinal movement of tower tops from design loads

FROM THE DESIGN LOADS OF THE FIVE LONGEST SUSPENSION BRIDGES

ES OF CABLES		(*c*) PROPERTIES OF STIFFENING GIRDERS				(*d*) PROPERTIES OF LATERAL SYSTEM				
						Horizontal Wind Force (Pounds per Linear Foot)				
Diameter (in.)	Total area[c] (sq in.)	No.	Type	Depth (ft)	Moment of inertia (all girders), in in.^{2} ft^2	Floor and vehicles	Cables	Total	Width of wind truss	Item
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	
36½	1,664	2	Trusses	25[f]	88,000	1,130	200	1,330	90	1
36	3,195	2	Chords	....	168	1,200	300	1,500	106	2
17½	383	2	Girders[e]	8	2,567	560	60	620	39	3
20½	1,050	2	Trusses	....	156,000	1,400	145	1,545	66	4
22	594	2	Girders[e]	11	5,860	800	120	920	74	5

Values shown are for the present single-deck condition. For the final double-deck condition, the weight will center of chords.

(*Continued*)

(*f*) DEFLECTIONS (FEET)										
	Vertical Deflection of Center Span[k]				% Grade					
Unit live load[j]	Load one-half center span		Load center span	Load side span	Maximum change	Maximum	Tilting from live load[l]	Lateral deflection of truss[m]	Longitudinal deflection tower[n]	Item
	¼ point	¾ point	Center	Center						
(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	
4,000	+12.41	−6.72	+7.34	−2.34	2.6	5.7	10.88	21.5	1.36	1
4,000	+5.88	−3.54	+2.92	−0.48	1.2	3.4	4.44	10.7	0.34	2
1,500	+10.95	−2.74	+10.50	−5.93	3.6	6.2	17.54	20.03	1.98	3
6,000	+10.70	−7.50	+11.85	−8.18	3.3	6.3	14.27	11.42	2.53	4
3,000	+8.41	−3.30	+6.50	−2.35	2.8	6.8	7.88	8.93	0.83	5

ut of bridge. [k] From live load at normal temperature; down deflections are positive. [l] Measured at the ction of the tower top from live load.

of five of the longest suspension bridges in the United States. Report (*a*) also includes a table giving the maximum center and quarter-point deflections for the same bridges, due to a uniform load of 200 lb per ft of bridge. This comparison seems of little value, since the uniform load per foot of bridge would cause varying loads on the cables in the different bridges due to the various widths of the bridges. Had the comparison been for a uniform load per foot of cable, the results would have been quite different. This table (not included in this report) shows that the vertical flexibility of the Tacoma Narrows Bridge was far in excess of any of the other bridges.

In Table 3 (from Report (*c*)), the torsional deformation of the five suspension bridges is given for a torsional moment of 10,000 ft-lb per linear foot of bridge, showing again that the Tacoma Narrows Bridge had far less resistance to deformation for a given torsional moment that any of the other bridges. A

TABLE 3.—COMPARISON OF TORSIONAL DEFORMATION OF TACOMA NARROWS BRIDGE WITH THAT OF OTHER BRIDGES[a]

Item	Bridge	Width[b] *b* (ft)	Unit load[c] *P*	Difference in Deflection (Ft)		Rotation (Radians)	
				¼ point[d]	Cables[e]	Actual	Ratio[f]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Tacoma Narrows:						
1	As built	39	2.564	3.18	8.154	0.2091	....
2	Fully braced design	39	2.564	0.296	0.760	0.0195	10.7 : 1
3	Toledo	59	1.695	0.26	0.441	0.0075	27.9 : 1[g]
4	George Washington	106	0.943	0.62	0.585	0.0055	38.0 : 1[h]
	Bronx-Whitestone:						
5	With stays	74	1.351	1.08	1.459	0.0196	10.7 : 1[h]
6	Without stays	74	1.351	1.22	1.648	0.0223	9.4 : 1[h]
7	Transbay	66	1.515	0.80	1.212	0.0184	11.4 : 1[h]
8	Golden Gate	90	1.111	1.36	1.511	0.0168	12.4 : 1[i]
9	Thousand Islands[j]	30.5	3.279	1.54	5.050	0.1656	1.3 : 1[k]
10	Deer Isle	23.5	4.255	2.92	12.424	0.5287	0.4 : 1[k]

[a] From Report (*c*), Appendix III, based on deflections at the quarter point of the loaded half of a span, due to a torsional moment of 10,000 ft-lb per foot of bridge, from one tower to midspan. [b] Center to center of cables. [c] Hundreds of pounds per linear foot of cable; up on the near cable and down on the far cable. [d] At the quarter point of the loaded half of the span, due to loads of 100 lb per linear foot of cable, from one tower to the midspan; up on the near cable and down on the far cable. [e] Difference in deflection between the two cables. [f] Ratio of the rotation of Tacoma Narrows Bridge to that of other bridges. [g] Based on data supplied by Shortridge Hardesty. [h] From Report (*a*), Appendix I. [i] Data supplied by Clifford E. Paine. [j] American crossing. [k] Data supplied by D. B. Steinman.

diagram is also given in Report (*a*) (Fig. 18, page 87) for the comparative tilting of the floors of these bridges based on a load of 100 lb per ft along one cable from one tower to the center and along the other cable from the center to the other tower. This diagram shows that the tilting of the Tacoma Narrows Bridge floor was much greater than that of the other bridges.

The ratios of torsional flexibility of the Tacoma Narrows Bridge to that of the other four large suspension bridges, based on vertical forces, is given in Col. 7, Table 3. In the words of Report (*a*), this comparison shows that the Tacoma Narrows Bridge was far more susceptible to torsion than any of the other bridges. This comparatively great torsional flexibility is due to a combination of exceptionally great vertical flexibility and unusually narrow widths and not to lack of torsional resistance of the floor structure, nor to the narrowness of the bridge alone.

Report (*a*) discusses the effect of increase in weight and concludes that to bring its comparative vertical rigidity within the range of the Golden Gate and Bronx-Whitestone bridges would require that the Tacoma Narrows Bridge be made 2½ times heavier than it was, other proportions remaining the same.

Report (*a*) also discusses the sag in the cables and concludes that a considerable decrease would have had little influence in increasing the rigidity of the Narrows Bridge.

The effect of stiffening trusses or girders is treated and the statement is made that in the Tacoma Narrows Bridge the girders were far too flexible since they

influenced the deflections of the unstiffened cables at the quarter points to the extent of only 1.3%. In other words, they were practically ineffective in stiffening the cable except locally. The stiffness of girders or trusses is increased most effectively and economically by increasing the depth. Diagrams are given to show the effect of the increased weight and stiffening of the girders on the vertical deflections of the Tacoma Narrows Bridge.

The report also considers the effect of the long side spans upon the flexibility of the center span and compares it unfavorably with the Bronx-Whitestone Bridge and the George Washington Bridge.

Finally, in discussing torsional and vertical rigidity, the report states that without further aerodynamic experiments and experiences on actual structures it is not possible to set a limit to torsional flexibility. Judging from the behavior of the Golden Gate Bridge, however, it would not seem advisable at this time to permit a torsional flexibility exceeding materially that of the Golden Gate Bridge. On this basis a width of 39 ft appears inadequate, and even a 53-ft width would require more rigid trusses than appear necessary for adequate vertical rigidity.

The effect of width, weight, and girder stiffness on torsional deformations is shown in a diagram and indicates that only an increase in the width to 53 ft, an increase in weight by 100%, and stiffening trusses 24 ft deep, can secure a torsional rigidity approaching that of the Golden Gate Bridge.

The report includes also chapters on "Aerodynamic Forces Acting on Suspension Bridges and the Oscillation Resulting Therefrom" and "General Conclusions," which give the arguments upon which the summary of conclusions previously given in this report were based.

There are also nine appendixes containing bibliography; copies of reports upon the failure; a mathematical treatment of the vibrations of suspension bridges by W. D. Rannie, graduate assistant, Guggenheim Aeronautics Laboratory, California Institute of Technology, at Pasadena; a report entitled "Experimental Investigations of the Aerodynamic Characteristics of the Suspended Structure of the Tacoma Narrows Bridge," by Louis G. Dunn, instructor in aerodynamics, Guggenheim Aeronautical School, California Institute of Technology; a report entitled "Observations of Motions of Golden Gate Bridge Wind Storms of February 9, 1938, and February 11, 1941," by R. G. Cone, M. Am. Soc. C. E., chief engineer; and a summary of Professor Farquharson's investigations which are discussed in Appendix IV of the present report.

APPENDIX II

Review of the Davis Report (*b*)

In addition to the Board of Investigation appointed by the Hon. John M. Carmody, another Board was set up by, and reported to, James A. Davis, Acting Director of Highways for the State of Washington. The members of this Board consisted of L. J. Sverdrup, M. Am. Soc. C. E., consulting engineer of

St. Louis, Mo.; Francis Donaldson, consulting engineer and Special Partner of Mason and Hangar Company, New York City; Russell G. Cone, M. Am. Soc. C. E., engineer of the Golden Gate Bridge and Highway District.

This Board was asked to report on the following matters:

(1) The extent of the loss occasioned by the bridge collapse, including the cost of repairing or replacing the property damaged and lost, with materials of like kind and quality, with proper deduction for depreciation;

(2) Whether the cables installed to correct the bridge sway shortly before the collapse contributed to the loss; and

(3) Whether the bridge foundations are in any manner affected by the collapse.

Part I of Report (*b*) was dated February 17, 1941, and gave the amount of loss as $4,297,098, based on the November 7, 1940, price of fabricated steel. It also made a supplementary report based on prices prevailing on March 1, 1941, amounting to $4,473,493. These figures were based upon detailed studies of the cost of the necessary dismantling and reconstruction, including rental value for equipment, overhead, and allowance for salvage. Part II was dated June 26, 1941, and discussed the causes of the failure.

The conclusions of this Board, as to the necessary repairs, are:

1. The main cables must be dismantled and removed;
2. Towers 4 and 5 must be dismantled to the pier tops;
3. The concrete embedment of the tower anchorages must be cut away so as to expose these anchorages for inspection, and to permit the replacement of certain anchorage steel known to have been severely overstressed;
4. A large part of the concrete in the cable anchorages placed subsequent to the spinning operation must be removed, to permit the restringing of the cables;
5. The spans of the west approach must be dismantled;
6. The upper section of Tower 3 must be dismantled; and
7. The main piers would be suitable for re-use after minor repairs to their tops and new anchorages installed for the towers, provided further measurements do not reveal any movement of the piers themselves.

Detailed reasons for these changes are given in the body of Report (*b*), but will not be included here since they have no bearing upon the cause of failure.

The conclusions of this Board relative to the design of the bridge (see Fig. 8) may be summarized as follows:

(*a*) The ratio of width to span was inadequate;

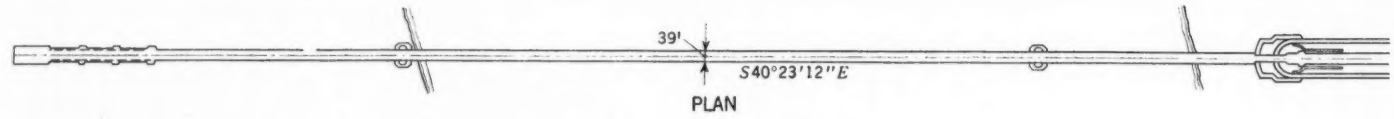
(*b*) The vertical stiffness was insufficient;

(*c*) The bridge details did not have sufficient capacity to dissipate energy and dampen the distortions caused by aerodynamic forces;

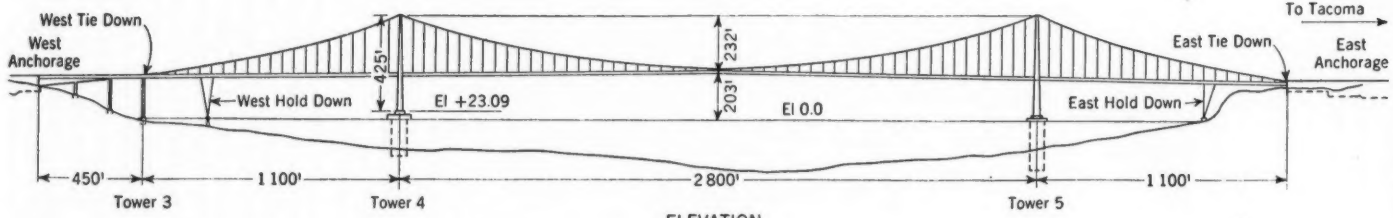
(*d*) The solid web stiffening girders were extremely sensitive to winds of low velocity; and

(*e*) The suspended structure lacked necessary torsional rigidity.

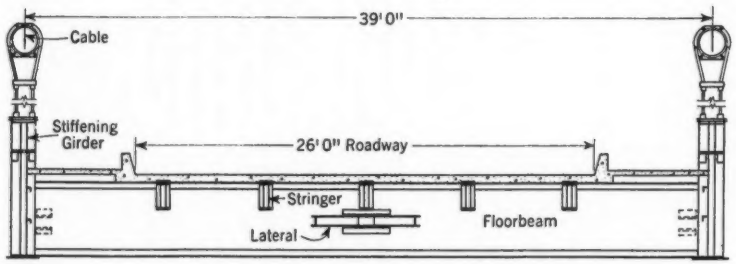
With respect to the devices installed to restrain oscillations, the report states that the hydraulic buffers at main towers were ineffectual; and the midspan



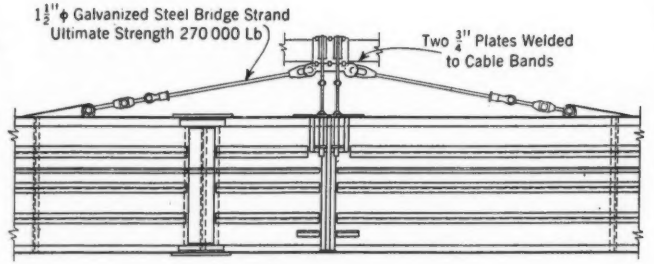
PLAN



ELEVATION



TYPICAL CROSS SECTION THROUGH BRIDGE



LONGITUDINAL SECTION AT CENTER MAIN SPAN

FIG. 8.—ELEVATIONS AND CROSS SECTIONS OF THE BRIDGE

diagonal stays, with their connections, were not strong enough to withstand the forces to which they were subjected and their installation was unfortunate.

Report (*b*) states that the behavior of the bridge was completely different from that of any other modern suspension bridge, except those bridges having solid girder stiffening systems that have been constructed recently; namely, the Thousand Islands, Deer Isle, and Bronx-Whitestone bridges. The report, however, fails to call attention to the fact that several other suspension bridge failures were marked by similar behavior, as, for example, the Brighton Chain Bridge which failed in 1836 and was fully described in the Reports of the Corps of Royal Engineers (see Appendix VI).

Other conclusions of the Board were that all known damping devices that had been studied and all devices deemed practicable had been installed, and that experiments with fairings and other suggestions to reduce the effect of aerodynamic forces that had been under way had to be abandoned because of the seizure by the government of the wind tunnel in which they were being made.

Other comments by the Board were to the effect that engineers concerned with other bridges that had developed undulations had never feared that these motions would cause serious structural failures, but were merely a nuisance to traffic and an undesirable source of wear on moving parts; also, that on all the other bridges, except Tacoma Narrows, the installation of backstays, inclined center stays, and end brakes had reduced the movement. By "all the other bridges" the Board presumably referred to the Thousand Islands, Deer Isle, and Bronx-Whitestone bridges, as it would appear from its report that the Board was not considering the failures, due to wind, of various suspension bridges as described in various articles referred to in the Bibliography given in Appendix V of the present report.

The following explanation of the behavior of the effect of the wind on the structure is given:

> "Beginning with the knowledge that the Tacoma Narrows Bridge was an extremely flexible and ribbon-like structure, we find that it was sensitive to the dynamic forces of steady moving wind stream, which caused undulations to start and to increase in magnitude. In addition it could be expected that certain gusty winds would cause the span to undulate. Since the floor was attached to the main cables at frequent intervals by means of the suspenders, all undulations of the floor were reflected in the main cables. Thus, any wind stream or gust which raised or lowered the floor also caused the cables to change their shape to conform. These changes in shape occurred in wave-like movements of various types. In certain of these wave-like movements and especially in the type having a single node at the center of the main span the cable sag at one quarter point was increased and the sag at the other quarter point was decreased. To assume such a shape the center point of the cables must move longitudinally. The longitudinal movement of the cables was resisted by the restraining influence of the short suspenders, but principally by the strength of the inclined guys at the center of the main span. The effect of these guys was to pull the suspended floor along with the cables. The movements here described were recorded not only by motion pictures but also by observations at the main towers. As the increment of sag change at the quarter points was reversed, the center point of the cable moved in the opposite direction and the floor was pulled along in the opposite direction. Thus we find in fact a compound pendulum

system that induced forces between the floor structure and the cables through the center guys. In addition to the force required to overcome the inertia of the floor there was additional restraint to this movement by reason of the hydraulic braking devices at the towers. The force exerted by the inertia of the moving cables was resisted by the diagonal stays attached to the stiffening girders. These stays were attached to the cable bands at the center of the span. The cable band castings resisted sliding along the cables

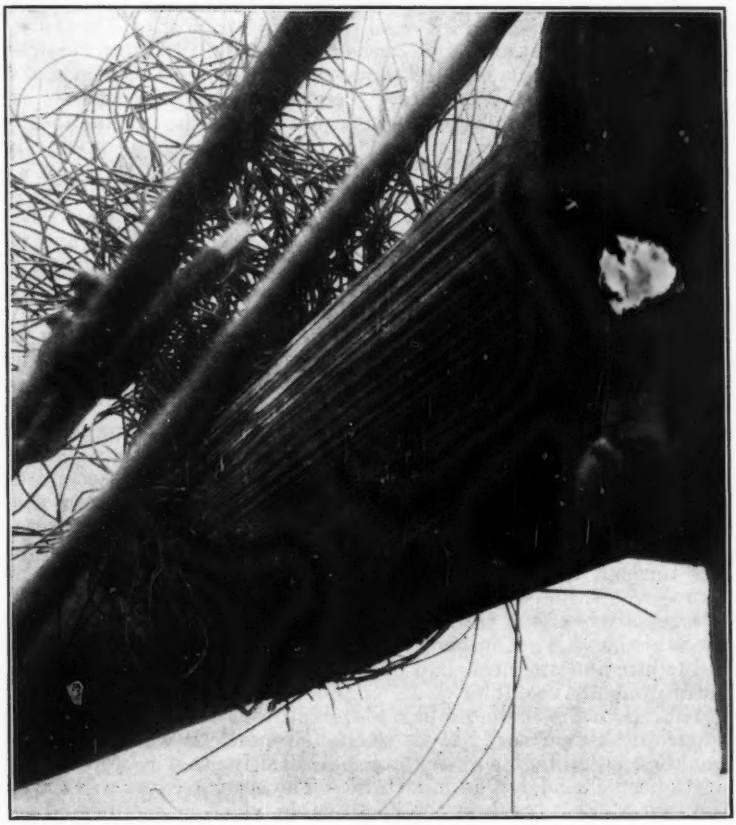


FIG. 9.—DAMAGE TO NORTH CABLE

by reason of the friction exerted on the cables. This friction is developed by the tension in the cable band bolts pressing the cable band castings against the cables. The weakest link in the inclined stay connection was the friction between the band and the cable. When it is considered that the cables are composed of a large number of individual wires and are slightly spongy in character, it can be seen that the readjustment of the relative positions of the wires would cause varying pressure of the cable bands on the cable resulting in variable frictional resistance to sliding. With the alternating pull on the cable bands the strength of the connections on the two cables became unequal. It follows that large forces would slip the cable having the weakest connection through the cable band. Actually, the north cable slipped through the cable band as deduced from the wreckage and in doing so broke a considerable number of cable wires [see Fig. 9].

Witnesses state that this slippage occurred before 10:15 a.m. or at the approximate time that the bridge developed a torsional motion.

"If we consider the longitudinal motion of the undulating suspended structure and cables with one mid-span guy holding and the other free on the other cable, we see that as the loose band of the north cable moves westward, the center of the north cable moves eastward with respect to the center of the opposite cable, and the sag of the east half of the north cable becomes greater than that of the east half of the south cable, thus tilting the east half of the suspended span from south to north. On the west half of the span the conditions are reversed and the floor tilts from north to south. As the loose band slips back all motions and tilts are reversed and the two halves of the span are alternately presented to the wind stream at angles that favor amplification of the rocking motion and the further absorption of energy. The wind acting on the two halves of the span quickly built up large torsional movements. The torsional resistance of the span was very low. The distortions were accompanied by stresses beyond the strength of the connections of the center panel of laterals and the resistance of the center panel of concrete deck. The laterals were torn loose and the deck panel broken and dislodged. As the deck panel was dislodged it fell into the Sound. The stiffening girders were buckled near the center and thus what little torsional strength the section had was lost.

"With the torsional resistance lost, the remaining halves of the structure continued to oscillate as the structure absorbed energy at every movement. The energy absorbed could only be dissipated through friction between the moving parts and hysteresis of the metal. The bridge did not have the capacity to dissipate the amount of energy that was being added by the force of the wind and the distortions increased in amplitude. As they increased, additional sections of the structure were torn loose and dropped into the Sound. As the weight of each piece was released from the cables they were violently tossed upward, the towers and side spans were put in motion, and further damage was done by buckling the towers, slipping the cables through the saddles and damaging the main foundations and the anchorages.

"An analysis of the actions of the structure subsequent to the slippage of the cable band is not important. The slipping of the cable band was the initial failure and after this had occurred the breaking up of the rest of the structure was unavoidable.

"Your Board is therefore of the opinion that the collapse of the bridge was attributable primarily to its slender proportions which combined with the solid girder stiffening system made it sensitive to wind forces which had hitherto been considered unimportant. The specific cause was the slippage of the center cable band on the north cable which was subjected to forces that it was not strong enough to resist."

APPENDIX III

Review of the Carew Report (*c*) and (*d*) Entitled "The Failure of the Suspension Bridge over Tacoma Narrows," with a Supplementary Report Entitled "Proposed Reconstruction of the Suspension Bridge over Tacoma Narrows"

A Board of Consulting Engineers was appointed to report upon the damage to the bridge by the Narrows Bridge Loss Committee, Paul Carew, Chairman, representing the twenty-two original insurers of the Tacoma Narrows Bridge.

This Board consisted of Clifford E. Paine, Hardy Cross, Shortridge Hardesty, Holton D. Robinson, and Wilbur M. Wilson, Members, Am. Soc. C. E.

The Board did not confine itself strictly to determining the damage done to the bridge but also gave an account of the building of the bridge and of the behavior of the bridge prior to and during its failure. It also gave a description of the failure of other suspension bridges due to lack of resistance to vertical and torsional wind forces. A valuable feature of the report is a presentation in general terms of theories underlying the design of suspension bridges with particular reference to the effect of wind forces. The report includes a discussion of the torsional resistance of suspension bridges and methods of increasing such resistance. Moreover, it describes the behavior of other suspension bridges built since 1930 and the methods of damping installed to reduce the oscillations in the Bronx-Whitestone, Thousand Islands, and Deer Isle bridges; all of which have shallow stiffening girders instead of trusses, and all of which developed oscillations of considerable amount. Report (*c*) also contains the following probable sequence of events during failure:

> "It is probable that the first member to fail was the west guy of the mid-span tie connecting the south cable to the south stiffening girder * * *. The failure of this tie immediately threw the cables out of phase and started a sudden, violent twisting motion. The cable band on the north cable, to which the mid-span tie was attached, began to slip on the cable, eventually moving back and forth through a distance of about three feet. This action continued for almost an hour before the span began to break up and drop."

There is included in this report a detailed description of the damage done to the bridge, recommendations for methods of repairs and estimates of the cost of restoring the structure to the condition existing before failure at prices in effect on November 7, 1940, at $1,581,195, and at prices in effect on June 1, 1941, at $1,796,190. The extent of the damage is presented in detail; laboratory tests of wires cut from the northern cable are included. A copy of the insurance policy covering direct loss or damage to the property is given.

In the supplementary Report (*d*) a method proposed to reconstruct the existing structure, using the existing towers and cables, is given in detail and the proposed design is stated to have the following characteristics as compared to the original construction:

1. An open deck or grid instead of a solid floor, thus reducing vertical wind forces;
2. Stiffening trusses through which the wind can blow instead of girders having solid webs;
3. Trusses three times as deep as the girders used in original design and with eighteen times the moment of inertia;
4. Two lateral systems, one in each chord plane;
5. Definite torsional resistance provided by the use of four planes of bracing together with transverse frames; and
6. A floor system restrained against longitudinal drift.

No estimate is given for the cost of this reconstruction. Both Reports (*c*) and (*d*) were dated June 2, 1941.

APPENDIX IV

Review of Report (*e*) of Experimental Investigations at the University of Washington

During the latter period of the construction of the bridge a dynamic model was built at the University of Washington under the direction of Professor Farquharson. This research was sponsored by the Public Roads Administration, the Washington State Highway Department, and the University of Washington; and a report was compiled covering all phases of these investigations and certain studies made subsequent to the failure of the bridge.

Also incorporated in Report (*e*) was a record of a rather extensive wind-tunnel investigation made under the same auspices and direction at the University of Washington. These wind-tunnel studies were confined to static tests on several models of the bridge as built and numerous proposed modifications. These tests were started several weeks prior to the failure and were continued with more attention to detail after the collapse. Fig. 10, compiled from Report (*e*), represents a small portion of the fairly voluminous aerodynamic data gathered in these tests.

Considerable time was devoted to a study of the effect of various arrangements of stay cables and the following statement is taken from the report:

> "*Effect of Changes in the Stiffness of Truss and Changes in Dead Load.* * * * when the dead load on the bridge remained practically constant and the moment of inertia of the stiffening truss was increased, the logarithmic decrement increased, but the increase was not significant, as it was necessary to increase the moment of inertia 22.8 times to cause the logarithmic decrement to double. ['Logarithmic decrement' is a mathematical term to express the rate of dying down of vibrations of a system.] Increasing the dead load had an opposite effect to that of increasing the stiffness, the logarithmic decrement varying inversely as the dead load. For instance, when the dead load on the model as built, was increased from 9.42 lb. per ft. to 14.84 lb. per ft., the logarithmic decrement decreased from 0.00599 to 0.0042.
>
> "*Effects of Auxiliary Hold-downs.* Attaching auxiliary hold-downs on the side spans had little effect on the rate of energy dissipation * * *. When the auxiliary hold-downs were in the most favorable position, the decrement was increased from 0.0055 to 0.0077, an increase of 40 per cent. Auxiliary hold-downs were actually installed on the bridge at approximately 300 feet from the main hold-downs.
>
> "*Effect of Tower Stays.* Stays from both the towers at roadway level to the main cables and those from the tower tops to the stiffening trusses had a favorable effect on the decrement, but to make them effective it was necessary to tighten them until the bridge was badly distorted. When they were placed in the most favorable position studied, which was the one where the stays in the main span were attached 4.0 feet (400 feet on the bridge) and those in the side span were attached 3.5 feet from the tower, it was necessary to tighten them until the truss below the point of attachment was raised or lowered 0.2 inches (20 inches on the prototype).
>
> "Stays from the tower to the main cable had the most noticeable effect on the decrement * * *. When they were in the most favorable position the decrement was increased from 0.0055 to 0.0256, an increase of 365 per cent.

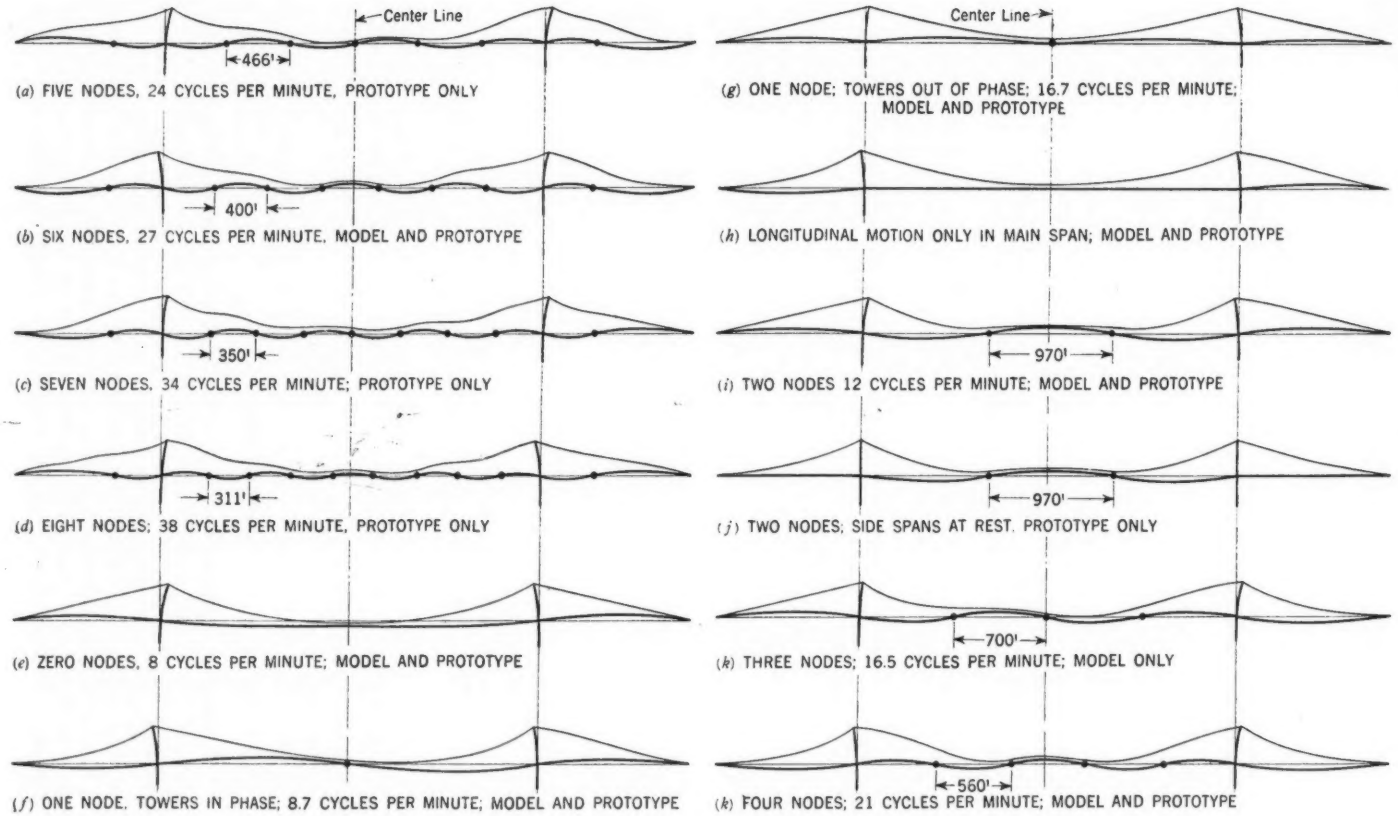

Center Line
466'
(a) FIVE NODES, 24 CYCLES PER MINUTE, PROTOTYPE ONLY
400'
(b) SIX NODES, 27 CYCLES PER MINUTE, MODEL AND PROTOTYPE
350'
(c) SEVEN NODES, 34 CYCLES PER MINUTE; PROTOTYPE ONLY
311'
(d) EIGHT NODES; 38 CYCLES PER MINUTE, PROTOTYPE ONLY
(e) ZERO NODES, 8 CYCLES PER MINUTE; MODEL AND PROTOTYPE
(f) ONE NODE, TOWERS IN PHASE; 8.7 CYCLES PER MINUTE; MODEL AND PROTOTYPE
Center Line
(g) ONE NODE; TOWERS OUT OF PHASE; 16.7 CYCLES PER MINUTE; MODEL AND PROTOTYPE
(h) LONGITUDINAL MOTION ONLY IN MAIN SPAN; MODEL AND PROTOTYPE
970'
(i) TWO NODES 12 CYCLES PER MINUTE; MODEL AND PROTOTYPE
970'
(j) TWO NODES; SIDE SPANS AT REST. PROTOTYPE ONLY
700'
(k) THREE NODES; 16.5 CYCLES PER MINUTE; MODEL ONLY
560'
(k) FOUR NODES; 21 CYCLES PER MINUTE; MODEL AND PROTOTYPE

FIG. 10.—MODES OF MOTION OBSERVED ON TACOMA NARROWS BRIDGE

"Stays from the tower top to the stiffening truss had a lesser effect * * *. When they were in the most favorable position, the decrement was increased from 0.0055 to 0.0172 or 210 per cent.

"When the side span and main span stays, instead of being rigidly fastened to the towers, were both attached to a rocker, the effect on the rate of energy dissipation was still more noticeable. Most of the oscillation-dissipation curves for this condition were difficult to analyze as they were very irregular, but an analysis of one of the most regular curves in which the stays were attached to a rocker at the tower top showed the decrement was 0.042, an increase of 740%. When friction was added at the rocker the decrement increased to 0.057, an increase of over 900% as compared to the model as built.

"*Effect of Tower Stiffness.* Fixing the towers at the roadway level had little effect on the decrement, increasing it from 0.0055 to 0.0065, or 20%. Fixing the towers at the top had a somewhat greater effect, increasing the decrement to 0.0172, an increase of 220%, while fixing the tower at both the top and at the roadway, increased the decrement to 0.0200, an increase of 270%.

"It must not be thought that the magnitudes of the logarithmic decrements for the prototype are the same as for the model. The rate of energy dissipation due to structural damping depends on the hysteresis of the material of which the structure is made and the friction at the connections; these differ in the model and prototype. If all the factors that affect the rate of structural damping of the prototype could be evaluated—in the present state of our knowledge this is not possible—it would be theoretically possible to build a model from which logarithmic decrements approximating those for the prototype could be obtained (of course the proper corrections would need to be made to the observed values as obtained from the model).

"Although the study indicates that the rate of energy dissipation of the model could be increased as much as 900 per cent, the rate of dissipation would be still very low, as compared with the rate of dissipation in a beam type bridge. The logarithmic decrement of 0.0055 for the model as built is less than 2% and the value of 0.057, the highest value found in the study, less than 20% of the accepted value of 0.3 for beam type bridges."

In connection with these laboratory studies a large number of observations were taken in the field. These observations extended over the active life of the structure and also recorded its death. Fig. 7 shows a few single frames enlarged from 16-mm film which are typical of many hundreds of feet of film obtained.

It has been noticed in studying wind failures of certain bridges in Great Britain that the character of the terrain may have influenced the wind forces on the structure and this may possibly have been the case at the Tacoma Narrows Bridge.

The following, with respect to the possible effect of this factor, is quoted from the University of Washington Report:

"As has been previously noted, the bridge was rarely observed to move in a northerly wind, while it almost invariably undulated in a southerly wind no matter how low the velocity. Static wind tunnel tests at the University of Washington have shown indications of aerodynamic instability when the direction of the relative wind falls within a degere or two of the horizontal.

"It has seemed reasonable to assume that the nature of the terrain which modifies the flow of the wind in the vicinity of the bridge is such that northerly winds impinged on the bridge at a relatively large positive or

negative angle. Examination of the location map [see Fig. 1] and the aerial view shown in [Fig. 2] show Point Defiance, with its tall bluff, projecting out into the waterway to the northeast of the bridge. Northerly winds obviously have to hurdle this obstruction and the wind may be assumed to acquire an undulating motion such as to cause it to impinge on the bridge with a large negative angle or possibly to be reflected from the surface of the water with a large positive incidence.

"A very tall smoke stack at the Tacoma Smelter is located just out of the field of view to the right of [Fig. 2]. On several occasions a heavy discharge of smoke from this stack has been noted as it assumed this undulatory pattern in flowing over the bluff. This bluff is something over two and one-half miles from the bridge and the smoke pattern does not remain intact long enough to demonstrate the flow at the bridge side, but evidence seems to indicate a vertical undulation in the flow for some distance down wind from the point.

"The approach from the south is very much clearer than that from the north and it is to be expected that a southerly wind would be relatively free from large disturbances. When the wind blows from the southeast there is an unobstructed approach over water of seven or eight miles. A wind directly from the south blows over a long plateau which discharges the wind over the water at about the level of the suspended structure.

"Occasionally the bridge would be found quite at rest in a strong southerly wind. Some time before preparations had been made for careful observation and before an anemometer was available a strong southerly wind estimated at nearly 20 mph blew for several hours without causing motion. This situation was unusual, however, and it can be reasonably stated that in general the bridge moved in a southerly wind and remained at rest in a northerly wind."

APPENDIX V

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APPENDIX VI

"XV.—A Short Account of the Failure of a Part of the Brighton Chain-Pier, in the Gale of the 30th of November, 1836. By Lieutenant-Colonel Reid, Royal Engineers.

"The same span of the Brighton chain-pier (the third from the shore), has now twice given way in a storm. The first time it happened in a dark night, and the storm was accompanied by much thunder and lightning: the general opinion of those who do not inquire into the causes of such matters was, that it was destroyed by lightning; but the persons employed about the pier, and whose business it was to repair it, were satisfied that the first fracture was neither caused by lightning nor by the waters, but by the wind.

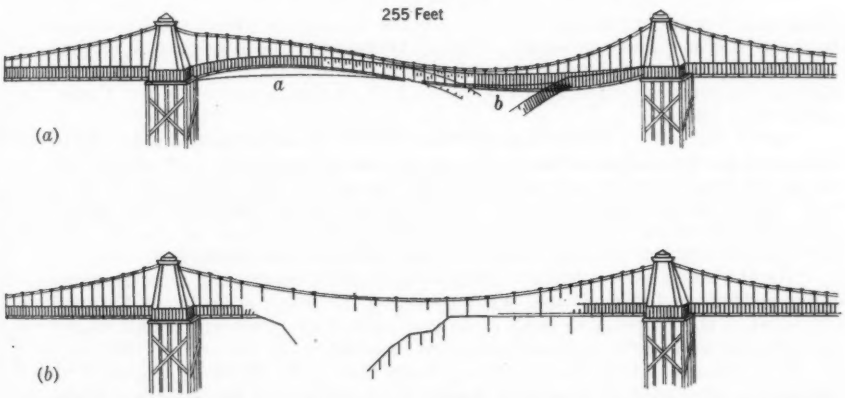

Fig. 11.—Destruction of the Chain Bridge at Brighton, England, November 29, 1836

"The fracture this year was similar to the former, and the cause evidently the same. This time, it gave way half an hour after mid-day, on the 30th of November, 1836, and a great number of persons were therefore enabled to see it.

"The upper one of the two sketches annexed, shows the greatest degree of undulation it arrived at before the road-way broke; and the under one shows its state after it broke; but the great chains from which the road is suspended remained entire. [Fig. 11 is an attempt to reproduce these sketches from available copy. In Fig. 11(*a*) the point a represents the undersurface of the roadway and point b the uppersurface, both of which were said to have been visible at the same time.]

"When this span became relieved from a portion of its load by the road-way falling into the sea, its two piers went a little on one side, and the curve of the chain became less, as in the sketch. The second and fourth spans in these sketches, are drawn straight, merely to show better the degree of undulation of the third span. These also undulated greatly during the storm, but not in the same degree as the third span. A movement of the same kind in the road-

way has always been sensibly felt by persons walking on it in high winds; but on the 29th of November, 1836, the wind had almost the same violence as in a tropical hurricane, since it unroofed houses and threw down trees. To those who were at Brighton at the time, the effect of such a storm on the chain-pier was matter of interest and great curiosity. For a considerable time, the undulations of all the spans seemed nearly equal. The gale became a storm about eleven o'clock in the forenoon, and by noon it blew very hard. Up to this period many persons from curiosity went across the first span, and a few were seen at the further end; but soon after mid-day the lateral oscillations of the third span increased to a degree to make it doubtful whether the work could withstand the storm; and soon afterwards the oscillating motion across the road-way, seemed to the eye to be lost in the undulating one, which in the third span was much greater than in the other three; the undulatory motion which was along the length of the road is that which is shown in the first sketch; but there was also an oscillating motion of the great chains across the work, though the one seemed to destroy the other, as they did not both, (at least as far as could be seen,) take place in a marked manner at the same time.

"At last the railing on the east side was seen to be breaking away, falling into the sea; and immediately the undulations increased: and when the railing on this side was nearly all gone, the undulations were quite as great as represented in the drawing.

"The ends of the joists of the road-way rest upon iron girders about one inch thick and five inches deep. They are made in lengths which are fixed together by oval bolts. The road-way seemed, therefore, to shake like a chain, and become, in fact, one with long links, and on examining the work after the storm, the girders appeared to have broken at the holes made for these oval connecting bolts.

"After the rail on one side gave way, (whilst the other remained,) the undulations of the two sides of the road-way became unequal, so that whilst half the *top* of the road was seen at one end of the span, half the *under part* of the road was visible at the other end, and this latter has been shaded in the sketch, to endeavour to shew it. Its motion was like that of a boat in a cross sea. It broke in five or ten minutes after the side-railway was destroyed.

"As the suspension rods are only hung up to the great chains by cast-iron caps, resting on these chains, without being fastened, some were thrown off: but most of the suspension rods, (which are about one inch and a half round rod iron) broke, and parts of them were seen hanging, as in the second sketch, from the great chains. Had the road-way been stiffened, either by a good trussed railing, or otherwise, it probably would have withstood this storm. From its oscillating side-ways as well as undulating along its length, it seems to require stiffening against both these motions. The rail which broke away, was only of upright bars; and there is no cross bracing to any part, excepting to the timber piles of the pier.

"The Brighton chain-pier consists of four spans of 255 feet each. It rests on piles, through which the waves roll, meeting with little resistance. The piles have stood well ever since they were loaded and the work finished; but previous to that, whilst the work was in progress, much difficulty was encountered from the piles being frequently drawn by the water floating them, and storms deranging them. The contractor who was engaged to execute the work, discouraged by the difficulties, gave it up; after which Captain Brown, of the Royal Navy, the projector of it, undertook to finish it and succeeded, the persons he employed being principally seamen. It is expected that the damage which has now occurred, will be repaired at the expense of 1000 *l.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PHYSICAL PROPERTIES THAT AFFECT THE BEHAVIOR OF STRUCTURAL MEMBERS

Discussion

BY WILBUR M. WILSON

WILBUR M. WILSON,[34] M. AM. SOC. C. E.[34a]—The action of a composite plate girder with silicon-steel flanges and a carbon-steel web has been presented in the discussion. The problem, which is within the scope of the paper, is one of considerable importance and would seem to justify a more detailed consideration than has been given. The following discussion is based upon the premise that a member with given geometrical characteristics buckles at a certain average critical strain rather than at a definite average critical stress. Actually, the buckling strength depends upon both stress and strain, but, for a stress range in which Hooke's law does not apply, the average strain at which buckling begins is much more nearly constant than the average stress. The discussion is based upon the further premise that the carbon-steel web of the composite girder will have the same thickness as the silicon-steel web of the all-silicon girder which otherwise might have been used.

The distribution of the stress and strain on a transverse section of a plate girder is shown by the diagrams of Fig. 5. To simplify the problem, the computations have been made on the basis of the gross section so that, for a girder symmetrical about the center line, the tensile and compressive stresses are equal. The girder is composed of a 120-in. by $\frac{3}{4}$-in. web, four 8-in. by 8-in. by $\frac{7}{8}$-in. angles, and six 18-in. by $\frac{3}{4}$-in. cover plates. Figs. 5(*a*) and 5(*b*) are for a section at which the maximum stress is 36,000 lb per sq in.; this is eight tenths of the yield point of silicon steel and corresponds to a capacity rating.

The area OACOC′A′O in Fig. 5(*b*) represents both the stress and the strain on a section of an all-silicon girder. The yield point has not been exceeded, and the stress is proportional to the strain over the entire section. The resisting moment corresponding to a maximum stress of 36,000 lb per sq in.,

NOTE.—This paper by Wilbur M. Wilson, M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Jonathan Jones; May, 1943, by A. B. Kinzel; June, 1943, by Almon H. Fuller; September, 1943, by Messrs. R. L. Moore, Leon S. Moisseiff, and Fred L. Plummer; and October, 1943, by A. A. Eremin.

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[34a] Received by the Secretary November 6, 1943.

based upon the gross section and computed by the moment-of-inertia method, is 28,373,400 ft-lb.

The area OACOC′A′O in Fig. 5(*a*) represents the strain on a section of a composite girder with a carbon-steel web and silicon-steel flanges. The same area also represents the stress on the silicon part of the girder, but the stress

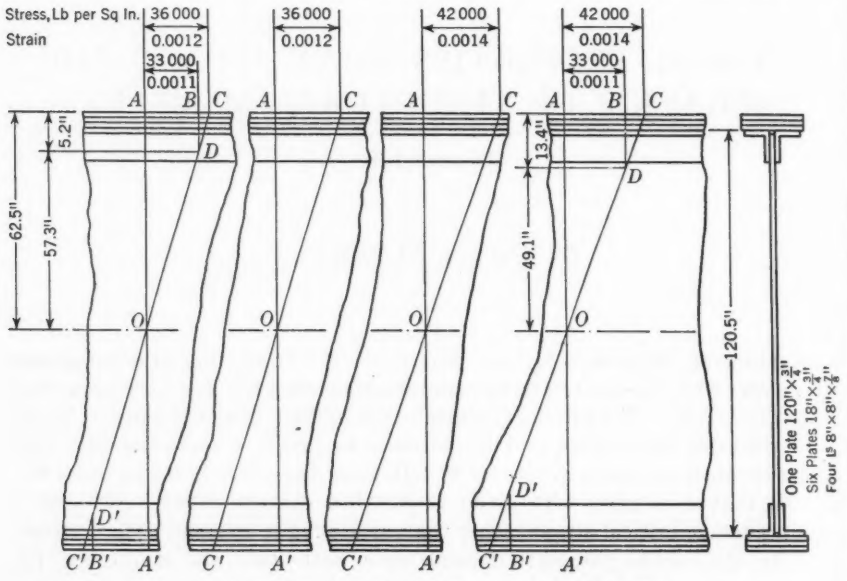

FIG. 5.—STRESS-STRAIN RELATIONS FOR VARIOUS GIRDERS

on the carbon web is represented by the area OABDOD′B′A′O. The resisting moment corresponding to a maximum stress of 36,000 lb per sq in., based upon the gross section and computed by the moment-of-inertia method, is 28,357,800 ft-lb, which is 99.94% of the resisting moment of the all-silicon girder. Although a small part of the carbon web between the angles is stressed beyond the yield point, the maximum strain in the web due to a moment of 28,373,400 ft-lb (the moment-resisting capacity of the all-silicon girder) is only 0.06% greater for the composite girder than it is for the all-silicon girder. Therefore, the composite girder with carbon web and silicon flanges would appear to have as great a moment-resisting capacity as the all-silicon girder when the maximum stress is 36,000 lb per sq in. Furthermore, there would seem to be no more danger of buckling and no more tendency for the rivets to loosen for the composite girder than for the all-silicon girder since both tendencies to fail are functions of the strain and there is no significant difference between the maximum strains for the two types of girders.

Consider now the conditions when the girder is subjected to an accidental maximum flexural stress of 42,000 lb per sq in.—a stress greater than any that would be considered in either the design or the rating of a bridge. The re-

sulting stress and strain are shown in Fig. 5(*c*) for the all-silicon girder and in Fig. 5(*d*) for the composite girder. Since no part of the all-silicon girder is stressed beyond the yield point, the strain and stress are both represented by OACOC′A′O; for the composite girder, the strain is represented by OACOC′A′O, but the stress in the web is represented by OABDOD′B′A′O. The resisting moment corresponding to a maximum stress of 42,000 lb per sq in. is 33,102,300 ft-lb for the all-silicon girder and 32,881,300 ft-lb for the composite girder, the latter being 99.15% of the former. Although this extreme increase in stress (to 42,000 lb per sq in.) has increased the part of the carbon web that is stressed to the yield point, the maximum strain in the web due to a moment of 33,102,300 ft-lb (the moment-resisting capacity of the all-silicon girder) is only 0.85% greater for the composite girder than it is for the all-silicon girder. Inasmuch as the tendency to buckle and the tendency to loosen rivets are both functions of the strain, these tendencies to fail are not significantly different for the two kinds of girders.

The statement has been made in the discussions that the writer did not answer the question which he raised in the last sentence of the "Synopsis." This question restated is: "Are there characteristics of the material that should be considered which are not now determined in acceptance tests?" The writer listed or implied in the "Summary and Conclusions" some characteristics of structural steels which are not now generally familiar to structural engineers but which he believes should become known before the steels are subjected to the services and fabrication methods that are becoming more common. This is particularly true for the newer steels, as stated by Mr. Jones.

Mr. Kinzel expresses the belief that the writer has been unduly conservative in his attitude toward low-alloy structural steels with special reference to the possible injurious effects of welding and flame-cutting. Mr. Kinzel is a metallurgist and is accustomed to addressing metallurgists. The writer is a structural engineer with a limited knowledge of metallurgy and is accustomed to addressing other structural engineers, many of whom likewise have a limited knowledge of metallurgy. Warnings to an expert in any field may be uncalled for whereas warnings to a novice in the same field may be entirely in order. Many structural engineers are novices in the field of metallurgy.

A conservative engineer is naturally reluctant to use steels that can be embrittled during fabrication by deviations from prescribed procedure. His problem is to be sure that, in the confusion of shop fabrication and field erection, the proper procedure is always followed. What is most to be feared is incidental welding and flame-cutting not called for on the drawing and done without the knowledge of the engineer and by other than approved procedure. The writer agrees with Mr. Kinzel that low-alloy steels can be flame-cut and welded satisfactorily when the proper procedure is followed, but he also believes that a word of caution intended to insure that the proper procedure is followed is in order. The warnings in the paper were to emphasize the need for caution and not to discourage the use of welding and flame-cutting or the use of low-alloy steels.

Mr. Kinzel's reference to the undesirable features of riveted structural members is quite in order. There are locked-up stresses in structures fabricated

by riveting. Punching and riveting do injure both carbon and low-alloy structural steels. Riveted members are particularly susceptible to fatigue failure as the writer's own tests indicate.[35]

With regard to embrittlement subsequent to plastic deformation, Mr. Kinzel states in the third paragraph of his discussion:

> "* * * It has been well demonstrated that fully killed steels—either the silicon or aluminum types—are not subject to this phenomenon. This fact removes a very large part of steel from consideration. Most of the low-alloy steel is fully killed. As to those steels which are not fully killed and may or may not exhibit strain aging, it should be stated that forming, punching, drilling, and similar processes used in general construction, and particularly in preparation for riveting, produce more than sufficient strain to cause the phenomenon, and such strain is set up at points of high stress concentration. Nevertheless, as Professor Wilson well demonstrates, experience justifies the conclusion that such steels are suitable for structural purposes after fabrication by riveting. Although one must be in accord with any plea for more information on any subject, it would seem that there is no need for the implication that further information on strain-age characteristics is needed vitally for immediate engineering use."

Mr. Kinzel does not include any mention of rimmed steels. Some rimmed steels are susceptible to embrittlement subsequent to plastic deformation. Structural specifications do not prohibit the use of rimmed steel. Unquestionably rimmed steel is used. One reason this mysterious (to the structural engineer) phenomenon should be investigated is that there have been occasional brittle failures of structural members in service for which there has been no acceptable explanation. Some years ago the writer tested fifteen wide plates without joints[36] (20 in. to 40 in. wide) to compare the unit strength of the wide plates with the unit strength of control specimens cut from the same parent plates. Of three supposedly identical specimens 20 in. wide, the elongation in 8 in. was 35.1% for one and 23.5% for another. The third broke with a brittle fracture with practically no elongation. The structural engineer rightly wants to know why these brittle fractures occasionally occur.

Mr. Kinzel asks how much experience would be necessary to determine whether or not any particular method of fabrication was susceptible to fatigue. Fatigue failures may occur after a few thousands of cycles or after a few millions of cycles. Except for temporary bridges, designs should be based upon a long life. Fatigue failures have developed in carbon-steel members with bad stress raisers after many years of service. If a railroad bridge is designed for 50 years of service, the fact that the members were intact after 30 years of service would not be proof that it was properly designed against fatigue.

In discussing the relative fatigue strength of low-alloy and carbon-steel structural members, Mr. Kinzel states:

> "* * * it is likewise true that, given reasonable surface conditions and geometry, fatigue strength does increase with the static strength in most instances."

Unfortunately, riveted structural members and welded structural members, except members with butt welds with the reinforcement ground flush, do not

[35] *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, Urbana, 1938, p. 103.
[36] *Bulletin No. 239*, Univ. of Illinois Eng. Experiment Station, Urbana, p. 11.

have "reasonable surface conditions" as judged by their ability to resist fatigue.

As a result of comparative fatigue tests of riveted joints in carbon, silicon, and nickel steels,[35] the Committee on Iron and Steel Structures of the American Railway Engineering Association revised its specifications for railway bridges, reducing the allowable stress in silicon-steel and nickel-steel members subjected to fatigue.[37] The present requirements may be somewhat more drastic than necessary but have been adopted until such time as more information is available.

Mr. Kinzel takes exception to the many cautions relative to low-alloy steels, flame-cutting, and welding. Not all of these have been answered in particular because, as stated previously, the differences between Mr. Kinzel and the writer seem to be in degree only and are due to differences in background and viewpoint. Although the writer believes that his cautions to structural engineers are in order, he believes that Mr. Kinzel's viewpoint is necessary to complete the picture and welcomes the discussion in which it has been presented.

The writer was quite gratified to note that Messrs. Moisseiff and Moore recognized the implication in the statement in the "Synopsis"—

> "It is reasonable to infer that, if certain physical properties are prescribed, there must be some quite definite relation between these properties and the behavior of structural members."

This statement does contain the implication that all of the physical properties prescribed may not be needed, at least not for all structural members. Specifications of a material are usually a compromise between what the engineer thinks he wants and what the manufacturer can produce at a reasonable cost. The structural engineer has been spoiled by the high ductility combined with a fair strength that is obtained in carbon structural steel, one of the cheapest structural materials available. When he began to seek a material of higher strength or of lighter weight, he was reluctant to accept a lower ductility. Conservativeness and inertia probably are at least partly responsible for the demand for high ductility in the high-strength steels.

The objective is shock-absorbing capacity of the structural member rather than ductility of the material; and, as stated in the paper, shock-absorbing capacity depends upon the geometrical characteristics of the member as well as upon the physical properties of the material. Also, as cited in the paper, there are structural members fabricated of steels of low ductility that have a greater shock-absorbing capacity than the usual type of member fabricated of A.S.T.M.-A7 steel. Yet geometrical characteristics of the member have few restrictions in the specifications, and, in so far as the literature indicates, have been given little consideration by the profession. Usually where the final objective is a function of two factors, as is true for the shock-absorbing capacity of a structural member, the characteristics of both factors are considered. Investigations to determine the interrelations of the ductility of the material and the geometrical characteristics of the member are greatly needed. A few exploratory tests have been reported.[38] Inasmuch as ductility does not affect

[37] "Specifications for Steel Railway Bridges," A. R. E. A., 1942, Pt. 3, Alloy Steels, paragraph 9 of the Foreword.

[38] *Journal*, Am. Welding Soc., July, 1943, p. 317-s.

the shock-absorbing capacity except as yield-point stresses occur, and the latter generally occur only under accidental loads, high-speed tension tests which simulate accidental shocks are needed. Also, because the geometrical characteristics are equally as important as ductility, tests on small specimens of constant cross section and smooth surface would be of little value in evaluating the behavior of a structural member. What would seem to be needed are tests of specimens with the geometrical characteristics of structural members and as large as it is possible to test.

Mr. Plummer asks the writer whether or not he considered it safe to use A.S.T.M.-A70 steel under certain conditions which he prescribed involving: Age embrittlement subsequent to plastic flow, multiaxial stress conditions, low temperatures, welding, and shock. What a collection of unsolved problems! If the writer had authoritative answers to all of the problems involved in Mr. Plummer's question, he would not have written this paper. As stated by Mr. Plummer, the changes that are taking place in structural engineering practice are so great that it is not safe to assume that old practices necessarily apply to new conditions. Moreover, the impatience of modern times makes it necessary to learn in the laboratory in a few years what, in the past, was learned to a considerable extent by experience over a long period of years.

The last paragraph of Mr. Eremin's discussion reads as follows:

> "In conclusion (2), Professor Wilson states that the plastic deformation permits the redistribution of uneven localized stresses. From Fig. 2, it is evident that the plastic deformation at the reduced sections, where the localized stresses occur, is considerably reduced. Therefore, its effect on the redistribution of stresses is negligible."

This is the action which the writer had in mind in the discussion of combined deformation stresses and load stress in which he stated (see heading, "Why Is Ductility Necessary: Errors in Workmanship (Item (3))"),

> "Likewise, a high, combined, load-and-deformation stress cannot be accepted in a tension member that has a reduced section at a point of maximum stress."

In discussing fatigue, Mr. Eremin states,

> "From Fig. 4 it is obvious that the fatigue effect on stress in the members may be estimated and considered as a special type of loading. Therefore, it is possible to design the structure for a combined loading including the fatigue effect."

It is the opinion of the writer that there are two distinct steps in the design of structural members subjected to fatigue. The allowable static stress has one value, which is constant for a given material. The allowable fatigue stress usually differs from the allowable static stress. Moreover, it is different for different types of members fabricated of the same steel and is different for different members in the same bridge due to the fact that some members are subjected to more stress cycles than other members of the same bridge. (Hip verticals are more susceptible to fatigue than end posts.) Likewise, the maximum stress for which the member is designed would be different in designing for static load than it would be in designing for fatigue. In designing a member

for static load, the maximum possible stress that could be produced by the design load should be used. In designing for fatigue, only the stress that is likely to occur frequently should be considered. To illustrate, consider a two-truss double-track bridge. There is a possibility that the two tracks will be loaded simultaneously so, in designing for static loads, both tracks should be considered loaded; but, because for most bridges the simultaneous loading of both tracks would occur at very infrequent intervals, only one track should be considered loaded in checking for fatigue. Of course, the empirical methods of designing for reversed stresses now in use do not take into account these possibilities, but it should be borne in mind that these empirical rules were written when there was relatively little known about the fatigue strength of structural members, as distinguished from the fatigue strength of small machined specimens. Tests have recently been made to determine the fatigue strength of structural members,[39] which probably will result in some revision of the paragraph in the specifications which control the design of structural members subjected to fatigue. Curves like the one shown in Fig. 4 are needed for the types of members fabricated of various materials. The data for some of the curves are available, but a large amount of testing remains to be done.

The best method of designing structural members subjected to reversals and repetitions of stress has yet to be determined. The number of structural members seriously affected by fatigue is relatively small. The design load is not the same for fatigue as for static stresses. The ratio of the allowable fatigue to allowable static stress is not the same for low alloys as for carbon steel; it is not the same for all types of members; and it is not the same for all members of the same structure.[40] It would seem to the writer, therefore, that the best way to design is to proportion the members for the static loads (dead load, live load, and impact), and then check those members subjected to repeated or reversed loads for the possibility of a fatigue failure.

The writer believes that the paper, together with the discussions which it has evoked, has served a useful purpose in that it has:

1. Emphasized the manner in which the ductility of the material redistributes the stress when static stresses reach the yield point in regions of stress concentration;
2. Pointed out conditions under which yield-point stresses are and are not dangerous;
3. Called attention to the fact that the final aim sought, large shock-absorbing capacity of a member, depends upon the geometrical characteristics of the member as well as upon the physical properties of the material;
4. Indicated the possibilities of developing new types of structural members with geometrical characteristics which will result in a large shock-absorbing capacity even though the material has less ductility than has generally been considered necessary; and
5. Enumerated research projects which might lessen the risk that would otherwise accompany a too rapid adoption of new materials and new fabricating processes.

[39] *Bulletins Nos. 302, 310, 317, 327,* and *344,* Univ. of Illinois Eng. Experiment Station, Urbana.
[40] *Bulletin,* A. R. E. A., Vol. 45, No. 440, p. 25.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOW AROUND BENDS IN STABLE CHANNELS

Discussion

BY S. M. WOODWARD, J. W. HOWE, AND C. J. POSEY

S. M. WOODWARD,[33] HON. M. AM. SOC. C. E., J. W. HOWE,[34] M. AM. SOC. C. E., AND C. J. POSEY,[35] ASSOC. M. AM. SOC. C. E.[35a]—The mechanism of flow around bends in open channels is extremely complicated, and the resultant difficulties of experimentation and observation have kept the number of published investigations small in spite of the pressing practical need for information. This paper contains a lucid description of the spiral motion and several other interesting phenomena, as observed in an experimental channel of adequate dimensions. Use of a simple rectangular channel permits comparison with the results of mathematical analyses based on different assumptions for the distribution of three components of velocity throughout the cross section.

According to the author, the fundamentals of flow around bends in open channels are best studied in the laboratory, rather than in natural streams, because of the irregularity and instability of the stream bed. That the first of these difficulties is not insurmountable has been shown by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield,[36] who made a detailed study of a full-sized river bend, and who observed several of the interesting phenomena discovered by the author in his laboratory investigation. Their report probably had much to do with convincing the skeptics referred to by the author.[8]

The most important advantages of the laboratory approach come from the comparative ease with which the main variables can be changed. Thus, otherwise identical runs can be made with different depth-width ratios, with different central angles, and with different ratios of radius of bend to width of channel. Although such a testing program would be lengthy and expensive even in the

NOTE.—This paper by C. A. Mockmore, M. Am. Soc. C. E., was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. J. M. Robertson, and Joe W. Johnson and M. A. Selim; and October, 1943, by I. Nelidov.

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[35a] Received by the Secretary October 23, 1943.

[36] "Flow Around a River Bend Investigated," by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield, *Civil Engineering*, May, 1934, p. 258.

[8] "Existence of Helicoidal Flow," by Herbert D. Vogel and Paul W. Thompson, *Civil Engineering*, July, 1934, pp. 370–371.

laboratory, it might yield answers to some of the important unsolved problems in this field.

The author's data show that in the bend tested the spiral was strong enough for bottom velocities to carry particles across the center line of the channel at an angle of nearly 45°. How does the "strength" of the spiral vary around the curve? Does it persist in the tangent downstream? How does its strength vary with variation in the dimensions of the bend and of the cross section? What is a satisfactory measure of the strength of the spiral motion?

The author's mathematical analyses both apparently assume that the spiral flow comes suddenly into existence at the point of tangency and remains constant around the bend. Replacement of this assumption with a more reasonable one would add considerably to the complications of the mathematical treatment, and yet would not permit a complete solution to be approached much more closely, for other factors, such as the vertical velocity distribution, still remain to be brought into agreement with observations.

Should the velocity distribution at the point of tangency be assumed to be the normal velocity distribution of the stream in a long, straight portion of its channel? Consideration of the water-surface profile through the beginning of the curve indicates that it is not normal—a deduction[37] made some years ago which is verified by the author's tests.

A question of great practical importance is that of the energy loss due to bends. A. A. Humphreys, Hon. M. Am. Soc. C. E., and Henry L. Abbot give the following formula, based on measurements on the lower Mississippi River:[38]

$$h = \frac{V^2 \sin^2 a}{134} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which h is the loss of head in feet, V is the velocity in feet per second, and a is a complicated variable, with only slight claim to rationality, introduced by L. G. Dubuat. It depends upon the radius, central angle, and width of the stream. Its computation is explained in detail by J. F. D'Aubuisson.[39]

An experimental study of a bend of the Iowa River, made in 1913–1914 by B. C. Boer and R. H. Urick, at Iowa University, Iowa City, showed that

$$\text{Loss of head} = \frac{0.0256\ V^2 \times \text{width of stream}}{\text{Geographic radius of curvature}} \quad \ldots\ldots\ldots (34)$$

On the basis of tests of the Tiger Creek flume, F. C. Scobey, M. Am. Soc. C. E., suggests that the value of Kutter's n be increased 0.001 for each 20° of curvature in 100 ft of flume.[40]

C. H. Yen found the head loss in a small 90° bend, rectangular in cross section and having an 11-in. width and 5-ft radius, to be $0.38 \frac{V^2}{2g}$.[41] His mea-

[37] "Hydraulics of the Miami Flood Control Project," by S. M. Woodward, *Technical Reports*, Miami Conservancy Dist., Dayton, Ohio, Pt. VII, 1920, p. 266.

[38] "Report upon the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, Lippincott, Philadelphia, Pa., 1861, p. 315.

[39] "Hydraulics," by J. F. D'Aubuisson, translated by Joseph Bennett, Little, Brown & Co., Boston, Mass., 1852, p. 215.

[40] "The Flow of Water in Flumes," by Fred C. Scobey, *Technical Bulletin No. 393*, U. S. D. A., December, 1933.

[41] "Effects of Channel Shape on Losses in a Canal Bend," by C. H. Yen and J. W. Howe, *Civil Engineering*, January, 1942, p. 28.

surements of secondary currents indicated that the velocity of the spiral flow was much higher at the bottom of the cross section than elsewhere. Modifications of the geometric proportions of the cross section resulted in large changes in the head loss.

Many features of flow around bends in open channels remain to be understood. Both laboratory investigations and measurements on natural streams will be necessary before the needed fundamental relationships have been formulated and evaluated.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS

Discussion

BY SILAS H. WOODARD

SILAS H. WOODARD,[13] M. AM. SOC. C. E.[13a]—Most of the critical discussions of the paper (nearly all are adverse criticisms which are not in agreement with each other) are flat rejections of the theory advanced in the paper because it does not agree with some other theory or because it does not fit the discusser's mental concept of the nature of stress. With one exception there is no citation of observed facts or recorded behavior to oppose the theory. The exception is the citation by Mr. Abrams of the relative compression tests of concrete cylinders ranging from 2 in. to 36 in. in diameter. However, the writer cannot accept those results as conclusive evidence: (1) Because the cylinders were tested with restrained ends and were not lubricated; and (2) because the material tested was concrete with coarse aggregate in the large cylinders and fine aggregate in the small cylinders. Even the mortar in concrete with aggregates 9 in. in diameter must have differed from the mortar in concrete with ½-in. maximum size of aggregate. Such differences would occur in relative workability or in water-cement ratio, or the density and workability of the entire mass would differ for different sizes of samples.

As to the nature of stress, the writer has canvassed enough engineers and physicists to learn that, although mental concepts of stresses are not all of the same pattern, his own are not entirely unique. The common practice of using the term "stress" as a ratio should not be confusing because it is still an expression of force per unit area. To be of use the computed stress should be a measure of distress. The engineer wants to know the margin of safety in his design.

For tensile stresses, the writer's concept is that, between each pair of adjacent particles of a solid, there is an elastic force which resists separation or

NOTE.—This paper by Silas H. Woodard, M. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by Messrs. C. M. Goodrich, Maurice P. van Buren, Joseph A. Wise, and David B. Hall; and June, 1943, by Messrs. Roberto Contini, I. Nelidov, Bernard L. Weiner, Richard W. Albrecht, Duff A. Abrams; September, 1943, by Messrs. C. A. P. Turner, and Mark W. Huggins; and November, 1943, by A. A. Eremin.

[13] Cons. Engr., New York, N. Y.

[13a] Received by the Secretary November 12, 1943.

nearer approach of the particles. So long as no other forces are acting, the distance between the particles remains constant, but other forces may be applied to separate the particles further. This applied force is resisted by the elastic force tying the particles together. If the forced separation becomes too great, the elastic resisting force reaches its limit and lets go; or, in other words, each particle passes outside the field of action of the resisting force.

When and if the physicists discover the exact nature of this elastic resisting force, the engineer will probably still rely largely on stress calculations.

The chief purpose of the paper was to emphasize the necessity for the use of Poisson's ratio in more refined stress computations. The distortion phenomenon expressed by Poisson's ratio is believed to induce stress not covered by the simple equation of external forces. The possibility of these induced stresses should not be dismissed simply by saying that they are inconceivable.

In an amorphous solid the particles are presumably arranged in a haphazard manner like marbles in a bag so that, if a force, tension or compression, is to travel from a point or particle on one side of the solid to an equal and opposite force on the other side of the solid, it must follow a zigzag course from particle to particle. At all the angle points of the zigzag line, components of force must be resisted either by external or internal force; if no external force exists, the resisting force must be internal. In the interior of the solid the components of adjacent zigzag lines, of course, will tend to balance each other, but not so at its surface. The net sum of all of the components of the principal forces or stresses is an induced stress normal to the direction of the principal stress.

If the principal stress is compression, the induced stress will be a tension normal to the principal stress and in that direction; the dimension of the solid will be increased following the law of Poisson's ratio; the particles in that dimension will be less closely spaced; and, if the limit of tensile strength is reached, they will separate, resulting in a tensile failure. This cannot be called inconceivable especially as brittle amorphous solids do behave that way, and the writer cannot conceive of particles being torn apart except by tensile stress.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A METHOD OF COMPUTING URBAN RUNOFF

Discussion

BY C. S. JARVIS, S. W. JENS, AND W. W. HORNER

C. S. JARVIS,[13] M. AM. SOC. C. E.[13a]—The extensive plottings, analyses, and calculations presented by the author should insure more than a casual consideration of his contribution. The orderly arrangement in logical sequence, step by step, facilitates the understanding of both the objective and the scientific approach. After a careful reading of the paper, the writer's interest centered on Fig. 12, showing the computed and actual runoff rates, the computed quantities being plotted against the observed.

It is certainly gratifying to find that most of the plotted points fall within a 20% tolerance zone. Without intentionally detracting from this accomplishment, it seems necessary to observe the extreme range of variation—for example, using the ordinate value of 1.5. The envelope of plotted values extends from abscissa value 0.65 to 2.15, showing an actual range from 0.43 to 1.43, representing 43% to 143% of the actual runoff. Similarly, for the ordinate 0.8, the range of computed values extends from 0.30 to 1.50, or from 37.5% to 87.5% of the actual value. Concentrating on the symbols representing data from Station 3, shown in Table 12, the range between envelope curves at the ordinate value of 0.45 extends from about 0.33 to 0.59, or from 73% to 131% of the observed quantity.

In view of these fairly wide dispersions, it seems appropriate to inquire as to the advisability of adopting an average factor, say 40% or 50% in Table 12, by means of which the computed peak runoff rates might be derived directly from the recorded rainfall rates, in the sixth column, under (*I*). Exercise of the same ingenuity and personal judgment required in assigning appropriate values for the shape and moisture factors in Table 12 should insure as close agreement between computed and actual peak runoff rates as was attained in either Fig. 12 or Table 12.

Fortunately, reports on research projects need not include the explorations that end in blind alleys, unless a brief account of such fruitless endeavor will

NOTE.—This paper by W. I. Hicks, was published in April, 1943, *Proceedings*.

[13] Cons. Engr., Washington, D. C.

[13a] Received by the Secretary October 28, 1943.

serve as a warning to others. It appears that full recompense in results of practical value has been realized for every operation thus far reported by the author. After setting up such definite forms and procedures, it is a simple matter to introduce refinements and revisions promptly after acquiring additional authentic data. Whether all new data shall be subjected to exactly the same processing or whether some abbreviated reductions shall be devised is relatively unimportant at this stage of development. However, with such a background of accomplishment as belongs to both the City and County of Los Angeles in the analysis and solution of flood and runoff problems generally, the conduct of operations from now on is practically assured as an example worth following under conditions largely comparable.

One of the greatest services that could be rendered by those inclined and equipped to make such elaborate analyses would be to render the results in the simple forms involving the proportion of rainfall amount as well as rate appearing with the runoff phenomena. Admitting that there is some reason for departing from the use of runoff coefficients and for adopting a more rigorously scientific approach, it does not seem to be too great a concession to those who have become accustomed to the simpler methods to translate results into the terms and expressions most familiar to them. Likewise, the use of some of the suggested scales for indicating the relative yields per unit of area would facilitate comparisons. Perhaps the author has some preferred method of rating the unit yield relation to drainage area, which would facilitate application of his method to much greater areas.

S. W. JENS,[14] Assoc. M. Am. Soc. C. E.[14a]—A very important contribution to the literature covering the application of the newer hydrology to practical design is contained in this paper. It is of particular significance as an indication of what a progressive municipal engineering organization can accomplish in the conduct and application of research to its design problems. The first few paragraphs of the paper present a very clear and succinct statement of the fundamentals involved.

The author rightly emphasizes the statistical meaning of the rainfall intensity-duration curves. It is easy to slip into careless thinking about this, with consequent erroneous use of the data. It seems preferable not to use the term "10-yr frequency storm" but rather "10-yr frequency rainfall rate" because the intensity-duration data merely give an average rainfall rate over the time of duration and may or may not represent the entire actual storm. In other words, the 10-yr, 30-min rainfall rate may actually be part of, say, a 60-min storm; and the 60-min average rainfall rate for that storm may be of a different frequency.

The techniques indicated by Mr. Hicks for the determination of the storm pattern and as a measure of the influence of antecedent precipitation are very interesting. Mr. Hicks states that the value of M determined by his studies applies to all rainfall frequencies. Can it be inferred that this value of M also applies to all durations? For areas having rains distributed throughout

[14] Partner, Horner & Shifrin, Cons. Engrs., St. Louis, Mo.

[14a] Received by the Secretary November 3, 1943.

the year, it is presumed that varying values of M would need to be developed to reflect properly seasonal differences in antecedent precipitation.

The considerable importance of the factor expressed by the percentage of impervious area directly tributary to the collecting system is given proper emphasis but, if possible for the particular areas in Table 1, it would be of value to have the average slopes indicated for overland flow, gutters, and drains. In discussing lag time the author (see Section IV, heading, "Derivation of Time Factors") states "It is dependent on the shape and distribution of impervious development of the drainage area * * *." Surface slopes seem to be fully as important a determinant.

Recent studies in this office of actual open channel recession curves show a straight-line relationship between discharge rate in cubic feet per second and volume of storage in cubic feet. In view of Mr. Hicks' findings that the same general relationship holds for the composite storage involved in urban drainage, it may well be that all ordinary recession curves can be expressed in some such form as:

$$q = K\, V^n{}_L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which q = rate of discharge or runoff in cubic feet per second or in inches per hour; V_L = volume of storage in cubic feet or in average inches of depth over the tributary area; K = a constant reflecting the physical characteristics of the area and the collecting system as well as the particular units being employed; and n = an exponent (found by the writer to be approximately 1). Additional similar study of rainfall-runoff data from a great number of different areas should be made.

In describing his technique of analyses, the author states that impervious surface loss has been experimentally determined as approximately 10% of the rainfall. It is difficult to understand why this should be so because analytical reasoning points to a fixed value for such loss for any particular surface. The amount of rainfall required to fill the undrainable depressions plus the surface film remaining after rainfall ceases would seem to be all the losses that can occur on impervious surfaces. Furthermore, both of these losses seem to be fixed amounts for any specific area, and all rainfall in excess of their sum should appear as runoff. It would be of interest if the author presented the experimental data that support his use of a constant percentage loss, irrespective of the rainfall intensity.

For routing a known inflow through a conduit to determine the effect of conduit storage, the storage equation has been found applicable. The following example illustrates the technique: For a peak inflow rate of 780 cu ft per sec, a time of 49 min and a hydraulic slope of $S = 0.0003$, with $n = 0.012$, a box section 13 ft wide with free flow 11 ft deep is indicated. This section is shown in Fig. 23 and was chosen to keep the excavation as shallow as possible to minimize the extent of the trench in wet ground. The section prevails for $l = 5{,}585$ ft and serves as an outfall to a pump station.

Fig. 23 shows also the hydrograph of inflow at the upper end of the box. For a short finite increment of time, Δt, the equation can be written: Average inflow rate times Δt equals outflow rate times Δt plus or minus the change in

storage in the time interval Δt. Let: I = average inflow rate in cubic feet per second for interval Δt; Q_1 = outflow rate at beginning of time Δt, in cubic feet per second; Q_2 = outflow rate at end of time Δt, in cubic feet per second; Δt = time interval, in minutes; V_{L1} = cubic feet of occupied volume in conduit

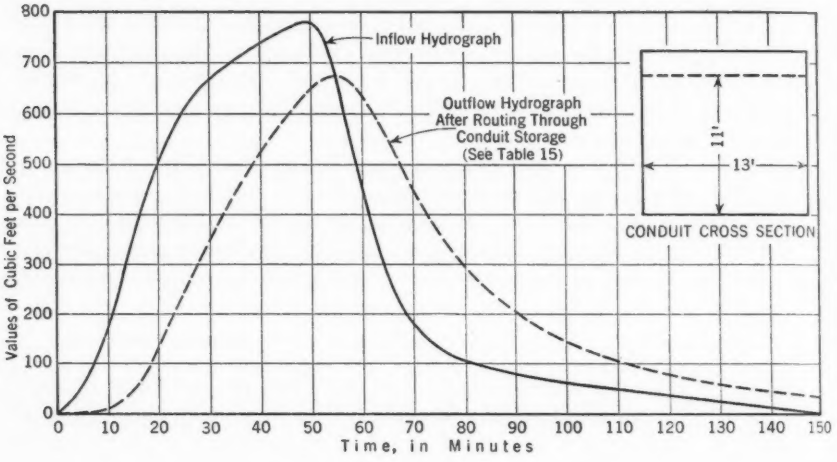

FIG. 23

at beginning of Δt; and V_{L2} = cubic feet of occupied volume in conduit at end of Δt. Then:

$$60\, I\, \Delta t = 60\, \frac{Q_1 + Q_2}{2}\, \Delta t + (V_{L2} - V_{L1}) \quad \ldots\ldots (8)$$

Rearranging terms, Eq. 8 becomes

$$fQ_2 = I + fQ_1 \quad \ldots\ldots (9a)$$

with

$$fQ_1 = \frac{V_{L1}}{60\, \Delta t} - \frac{Q_1}{2} \quad \ldots\ldots (9b)$$

and

$$fQ_2 = \frac{V_{L2}}{60\, \Delta t} + \frac{Q_2}{2} \quad \ldots\ldots (9c)$$

Table 14 gives the computations for the storage function curves shown in Fig. 24. Storage in the conduit has been assumed equal to the volumes under hydraulic gradients parallel to the flow line for varying depths of flow. This is not rigorously true, but, since change in storage is wanted, it is believed that the resulting curve represents the discharge-storage relationship with practical accuracy. The Manning formula has been used for discharge computations with n assumed as 0.012 for monolithic concrete. To catch the peak outflow more accurately, a curve for Δt equal to 1 min has been developed; for the remainder of the routing Δt = 5 min is used. As will be shown in describing the routing procedure, it is necessary to plot only the curve for fQ_2.

Table 15 gives the routing computations. Cols. 1 and 2 are taken from the inflow hydrograph. It should be noted that Col. 2 is the average inflow rate

TABLE 14.—COMPUTATION OF STORAGE FUNCTIONS

Depth d (ft)	Area A (sq ft)	Wetted perimeter (ft)	Hydraulic Radius (Ft)		Flow (Cu Ft Per Sec)		$\Delta t = 5$		$\Delta t = 1$	
			R	$R^{2/3}$	Q	$\frac{Q}{2}$	$\frac{S}{300}$	fQ_2	$\frac{S}{60}$	fQ_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	26	17	1.53	1.33	74.1	37.0	484.3	521.3	2,420.2	2,457.2
4	52	21	2.48	1.83	208.5	104.3	968.6	1,072.9	4,840.5	4,944.8
6	78	25	3.12	2.14	357.4	178.7	1,452.9	1,631.6	7,260.8	7,439.5
8	104	29	3.59	2.35	523.0	261.5	1,937.2	2,198.7	9,681.0	9,942.5
10	130	33	3.94	2.50	695.6	347.8	2,421.5	2,769.3	12,101.0	12,449.1
11	143	35	4.09	2.56	785.0	392.5	2,663.6	3,056.1	13,311.4	13,703.9

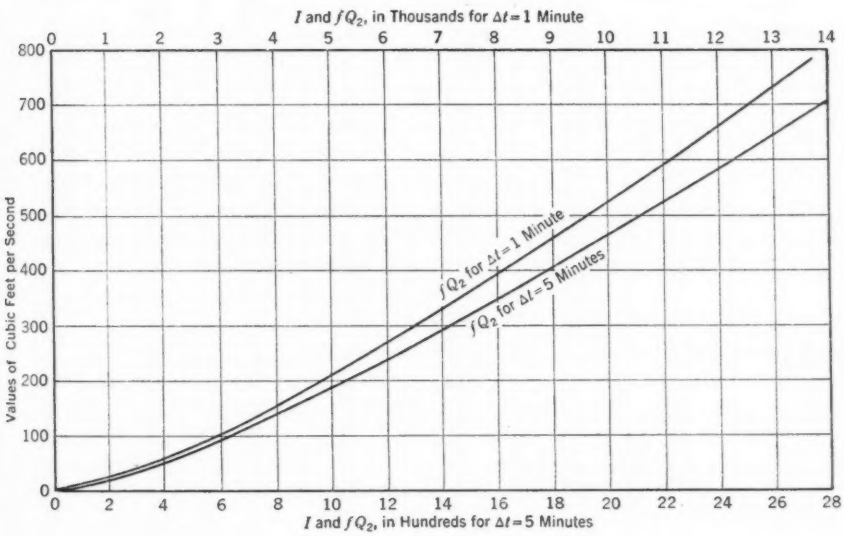
I and fQ2, in Thousands for Δt = 1 Minute
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
800 700 600 500 400 300 200 100 0
Values of Cubic Feet per Second
fQ2 for Δt = 1 Minute
fQ2 for Δt = 5 Minutes
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
I and fQ2, in Hundreds for Δt = 5 Minutes

FIG. 24

for the period and that Col. 3 is the outflow rate at the end of the time interval read from the proper storage curve. These outflows are obtained as follows:

(*a*) The first average inflow is laid off on the horizontal scale and the outflow rate at the end of the first 5-min period is read on the vertical scale (using, of course, the fQ_2 curve for $\Delta t = 5$); and

(*b*) The value shown in Col. 7 for the preceding line is placed in Col. 4; to this is added the inflow for the period under consideration (Col. 5); then the outflow arrived at in the previous interval is inserted in Col. 6 and this is subtracted, resulting in the value given in Col. 7. This latter figure is laid off on the horizontal scale and the outflow read on the vertical.

The process is repeated throughout the hydrograph and is merely a combined graphic and arithmetic solution of the storage equation. Transition from a curve for one-time interval such as 5 min to that for another Δt is accomplished

TABLE 15.—ROUTING COMPUTATIONS (UNITS, CUBIC FEET PER SECOND)

TIME, IN MINUTES		Average inflow	Outflow at end of interval	Col. 8[a]	+ Col. 3	− Col. 4[a]	= fQ_2 (see Fig. 24)
From	To						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	5	17	2	0	17	0	17
5	10	100	8	17	100	2	115
10	15	260	45	115	260	8	367
15	20	430	128	367	430	45	752
20	25	557	237	752	557	128	1,181
25	30	642	345	1,181	642	237	1,586
30	35	687	444	1,586	687	345	1,928
35	40	722	525	1,928	722	444	2,206
40	45	750	594	2,206	750	525	2,431
45	46	765	607	11,030	765	594	11,201
46	47	769	618	11,201	769	607	11,363
47	48	776	628	11,363	776	618	11,521
48	49	777	637	11,521	777	628	11,670
49	50	777	647	11,670	777	637	11,810
50	51	774	656	11,810	774	647	11,937
51	52	760	665	11,937	760	656	12,041
52	53	741	667	12,041	741	665	12,117
53	54	715	673	12,117	715	667	12,165
54	55	678	673+	12,165	678	673	12,170
55	56	640	672	12,170	640	673	12,137
56	57	600	666	12,137	600	672	12,065
57	58	558	658	12,065	558	666	11,957
58	59	510	647	11,957	510	658	11,809
59	60	467	636	11,809	467	647	11,629
60	65	347	548	2,568	347	636	2,279
65	70	215	449	2,279	215	548	1,946
70	75	150	362	1,946	150	449	1,647
75	80	118	295	1,647	118	362	1,407
80	85	97	244	1,407	97	295	1,209
85	90	86	206	1,209	86	244	1,051
90	95	76	170	1,051	76	206	921
95	100	67	144	921	67	170	818
100	105	58	124	818	58	144	732
105	110	51	106	732	51	124	659
110	115	45	92	659	45	106	598
115	120	40	80	598	40	92	546
120	125	34	71	546	34	80	500
125	130	28	62	500	28	71	457
130	135	22	55	457	22	62	417
135	140	16	48	417	16	55	378
140	145	10	42	378	10	48	340
145	150	4	35	340	4	42	302
150	155	....	29	302	0	35	267
155	160	....	25	267	0	29	238
160	165	....	21	238	0	25	213
165	170	....	18	213	0	21	192
170	175	....	16	192	0	18	174
175	180	....	14	174	0	16	158
180	185	....	12	158	0	14	144
185	190	....	11	144	0	12	132

[a] From the line preceding a given time interval.

by selecting an abscissa value for the new Δt representing the same outflow value (Col. 3) as was obtained for the prior Δt; the procedure is then reversed upon changing back to the original Δt.

The foregoing computation suggests that, because of the peak reduction caused by the storage effect, the box section, 11 ft high by 13 ft wide, required for the inflow peak of 780 cu ft per sec could be replaced by an 11-ft by 12-ft

box. A check routing through the smaller section is necessary to make sure that the lesser total storage accomplishes the requisite peak reduction.

Similar analyses were made of the storage effect for several other lengths and conduit sizes with various inflow peaks. These are shown in Table 16.

TABLE 16.—RESULTS OF ROUTING THROUGH CONDUIT STORAGE

No.	Cross Section (Ft)		Hydraulic slope	Length (ft)	Peak Flow (Cu Ft Per Sec)		Time[a] (min)
	Height	Width			Inflow	Outflow	
1	6.0[b]		0.0009	850	127.3	127.3	30.6
2	7.5[b]		0.00056	840	181.0	181.0	33.7
3	8.0[b]		0.00055	1,040	215.4	213.0	37.1
4	9.0[b]		0.00055	900	297	295	41.1
5	10.0[b]		0.00047	1,020	361	358	44.3
6	9.0	8.0	0.00039	1,020	332	327	47.4
7	9.0	8.0	0.00048	1,250	369	361	86.3
8	9.0	8.5	0.0005	1,470	430	421	90.2
9[c]	11.0	13.0	0.0003	5,585	780	673	49.0
10[d]	10.0	12.0	0.00037	5,650	1,375	1,361	117.2

[a] Time to peak inflow. [b] Circular cross section. [c] See Table 15 and Fig. 23. [d] Two sections, each 10 ft high by 12 ft wide.

These conduits all are in flat river bottoms and would be expected to exhibit more than ordinary storage effects. It seems significant, however, that only the extremely long, large box with a relatively short inflow hydrograph (49 min to peak inflow) gives an appreciable storage reduction. This is understandable from the viewpoint of the ratio of inflow volume to the available storage volume. The longer time hydrographs with greater volumes of inflow are more likely to fill the conduit before the occurrence of the peak.

All of the foregoing leads to the conclusion that the effect of conduit storage must be evaluated for each specific reach, and it is difficult to understand how reductions accomplished by any particular system can be applied as indicated by the author to any other system with different conduit sizes, lengths, and times.

The author has kept to the basic fundamentals throughout his development of practical tools for use in urban storm sewer design. His fine paper should encourage others to analyze rainfall-runoff records in the light of the hydrologic knowledge of the past few years.

W. W. HORNER,[15] M. AM. SOC. C. E.[15a]—Within the limits of his allotted space, Mr. Hicks has attempted to present fundamental data that have been derived over a period of years; and he treats a wide variety of urban storm drainage design situations. As a result, the paper is too condensed to give the reader an adequate picture of the design methods developed by the Bureau of Engineering of the City of Los Angeles. It is extremely interesting to note that Mr. Hicks and his associates had anticipated the design procedure which the writer and Mr. Jens outlined in a previous paper.[16]

[15] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

[15a] Received by the Secretary November 3, 1943.

[16] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.

The writer has read Mr. Hicks' paper carefully several times and has discussed it, but he may not have fully appreciated all of the elements involved in the procedure. Accordingly, it would seem desirable for him to outline his understanding with the request that Mr. Hicks clarify certain of the matters that were presented much too briefly in the paper.

Mr. Hicks and his associates have apparently analyzed a great many actual runoff hydrographs in the Los Angeles storm drainage system, and have found that they could correlate certain of the parameters in a manner which would permit the preparation of a base hydrograph of correct form for any particular time of concentration.

Separately, extensive probability studies have been made resulting not only in the duration-intensity diagram of Fig. 1, but also in the determination of the most probable pattern of variation of rainfall intensity. The use of these studies permits the plotting of gross mass rainfall as indicated in Fig. 3.

The curve of mass rainfall is then modified through infiltration, surface detention, gutter storage, and finally conduit storage to a diagram of mass runoff in conduit, from which the runoff rate hydrograph can be derived. The writer understands that a large number of such hydrographs have been prepared and may be modified by the methods described to produce the actual hydrograph of runoff for any set of specific conditions.

From the study of actual hydrographs and a correlation with the basic hydrographs, the author has been able to derive Eq. 5 and finds that this equation may be used satisfactorily as a basis for design in many instances. In actual practice it is understood that he uses either the summation of the derived hydrographs or Eq. 5, depending on certain limiting conditions.

From the foregoing, it would appear that, as the result of research work, the present actual design procedure may not be much more difficult or cumbersome than the sound application of the old rational method. If so, this answers one of the questions commonly raised; namely, can the engineer afford to take the time to evaluate infiltration, surface detention, gutter storage, etc., in the course of typical design practice. Apparently Mr. Hicks' research work makes it possible to evaluate all of these pertinent factors in an orderly manner and to reduce the procedure to a satisfactory working standard.

The writer would particularly like to have a more extended explanation of the moisture factor M. In part this factor as presented in Eq. 1 parallels somewhat the procedure, which the writer described,[17] in which he derived a moisture factor and gave it the same symbol M in order to relate infiltration capacity to antecedent precipitation. However, Mr. Hicks has introduced an evaporation factor which is fundamentally desirable. His value E apparently is not an evaporation rate, but some inverse function if the moisture factor is to increase with E. Further explanation of this factor would be desirable.

In recent design practice in the writer's office, particularly for the drainage of large airports, pattern storms or "design storms" have been derived very much in the manner Mr. Hicks describes. These are set up in hydrograph form and are routed successively from rainfall rate through surface detention to

[17] "Rôle of the Land During Flood Periods," by W. W. Horner, *Proceedings*, Am. Soc. C. E., May, 1943, p. 665.

gutter inflow; then through gutter storage to inlet inflow; and finally through a process of routing through conduit storage and offsetting hydrographs to a determination of peak rates along the drainage lines. The writer has found, just as Mr. Hicks did, that, after a number of such designs had been prepared, correlations could be perfected which would give satisfactory, approximate values without going through the routing process for each particular subsection of the system.

Mr. Hicks is to be commended greatly for the fundamental work that he describes. This indicates that one city engineering department is now in a position to design storm drainage on the basis of a full evaluation of infiltration and detention of all classes.

Corrections for *Transactions:* In April, 1943, *Proceedings*, page 462, line 17, change "clock time" to "elapsed time for design purposes"; in Fig. 3, change "V_{ro}" to "d_o," and in right-hand ordinate caption, change "=" to "−"; in caption for Fig. 9(*b*) change $\frac{\text{"}\Delta D\text{"}}{\Delta q}$ to "t_c"; in Fig. 10, left-hand ordinate caption change "=" to "−"; in Table 10(*a*), $t_c = 10$, under clock times of 110 and 120 minutes, change "0.45" and "0.56" to "0.57" and "0.36," respectively; in Table 10(*b*), $t_c = 10$, under clock times of 110 and 120 minutes, change "22.4" and "14.0" to "22.8" and "14.4," respectively; change Fig. 16, left-hand ordinate caption to read "elapsed time, in minutes"; in Fig. 18, change "V_{ro}" to read "d_o"; in Fig. 19, change "$T\,x$" to read "$t\,x$," and change the abscissa caption to read "elapsed time in minutes"; and, on page 491, change line 8 to read "Tables 6, 7, and a table of inlet times,[1a] and Figs. 2, 5, 13, and 14, are used."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEISMIC SUBSURFACE EXPLORATION ON THE ST. LAWRENCE RIVER PROJECT

Discussion

BY E. R. SHEPARD AND REUBEN M. HAINES

E. R. SHEPARD,[14] ESQ., AND REUBEN M. HAINES,[15] ASSOC. M. AM. SOC. C. E.[15a]—The question of disturbance of detectors in moving water has been raised by Mr. Moore. Some disturbance was observed in relatively still water where the detectors were supported by iron pipes projecting above the water surface. This appeared to be the result of surface wind ripples against the supporting pipes. With the tripod type of detector support, where the detector and supporting structure were entirely submerged, the disturbance, even in flowing water, was surprisingly small.

The writers are indebted to Professor Leet for clarifying the record with respect to credit for the equations used in computing overburden depths. The paper does not refer specifically to the origin of the formulas but states that they are given by Messrs. Ewing, Crary, and Rutherford. They, in turn, do not claim originality but refer to the formulas as the standard method of interpreting refraction data. Reference was made to this article because the equations there appear in a simple and usable form and are readily available for reference.

Reverse profiles, as described by Professor Leet, were actually obtained over the 80-ft distance between outside detectors. This 80-ft spread was a common path for waves through the rock from reverse shooting distances greater than the critical distance, beyond which the first arrivals are through the rock stratum. From such shots it was possible to compute apparent and true velocities in the rock directly under the 80-ft detector spread. It is true that the local velocity so obtained was assumed to prevail throughout the entire length of the seismic line. The small errors that might result from this assumption are usually minor in comparison with those referred to by Mr. Lee,

NOTE.—This paper by E. R. Shepard, Esq., and Reuben M. Haines, Assoc. M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by R. Woodward Moore, Esq.; and June, 1943, by Messrs. Arthur J. Grier, F. W. Lee, L. Don Leet, and William L. Shannon and Winthrop A. Wells.

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[15] Engr., Corps of Engrs., U. S. Army, Washington, D. C.

[15a] Received by the Secretary October 25, 1943.

involving small differences in the thicknesses of the slow beds which can introduce large time errors when applied to the same ray path in the fast beds. Because of this almost universally prevalent condition only one depth of overburden was recorded for each seismic line. The recorded depth was designated as that at the center of the line where the velocity profile in the overburden was determined most accurately.

In refraction shooting where a plurality of detectors is employed and a relatively few shots placed within the detector spread, the reverse profile through the fast beds is obtained only over a limited central portion of the line, the length of the common path depending on the depth of the overburden. If the overburden is deeper than a critical value depending on the shooting distances and detector spacings, it is possible that no common path will exist, and in such an event it would be impossible to compute the true velocity in the rock from the observed apparent velocities.

Mr. Grier describes a new and interesting modification of the well-known four-terminal electrical method of exploration. From a study of his circuit it appears that what the instrument actually does is to compare the potential drop $P_1 P_2$ with that of $P_2 P_3$. With a symmetrical distribution of the field current between C_1 and C_2, the potential ratio will be a measure also of the ratio of resistances $P_1 P_2$ and $P_2 P_3$. Mr. Grier does not say how far he places the current electrodes from the potential cups, but it is evident that, to insure a reasonably symmetrical distribution of the current in the ground, the current electrodes should be spaced equally from the respective potential cups P_1 and P_3 and at a sufficient distance from them to avoid the distorting effects on the field current of local variations in the resistivity of the top soil. These conditions are usually met, with the standard four-terminal arrangement of electrodes.

The writers are familiar with the seismic programs described by Messrs. Shannon and Wells. In Table 2, showing comparisons of seismic interpretations with drilling records, eight of the drill holes listed were not extended to rock. The only point at which the seismic and drill data appear to be seriously out of agreement is at the West Henniker Dam site where the drill reached the top of rock at a depth of 73.1 ft, and computations from seismic data gave a depth of 137 ft. The original records on this project have been consulted. The drill log for hole No. 9 reveals a soil profile as follows:

Depth, in ft	Soil
0 to 1	Top soil
1 to 11	Light gray, compact, silty fine to medium sand, with a trace of gravel
11 to 53	Grayish brown, compact, slightly gravelly, silty sand (glacial till)
53 to 70	Brown, compact, slightly silty fine to medium sand
70 to 72.6	Dark gray, compact, gravelly sand
72.6 to 89.3	(Bottom of boring) mica-schist, badly weathered to bottom of boring, over-all core recovery 32.5%

The seismic data at line 4 show the following velocity profile:

Depth, in ft	Wave velocity, in ft per sec
0 to 4.5	1,180
4.5 to 17.5	2,240
17.5 to 137	7,000
>137	16,550

It is evident that the velocity of wave propagation in the till and that in the badly weathered mica-shist were so nearly alike that it was impossible to discriminate between the two formations. No doubt other minor discrepancies shown in the table can be attributed to the same condition. It can be said for the seismic method that the weathered mica-shist, which is obviously unfit as foundation material for a masonry dam, did not register as sound rock. This instance is an excellent example of the limitations in geophysical methods of exploration and emphasizes the need for experience and sound judgment in the interpretation of all geophysical data.

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Founded November 5, 1852

DISCUSSIONS

RÔLE OF THE LAND DURING FLOOD PERIODS

Discussion

BY WALDO E. SMITH, P. B. ROWE, AND C. S. JARVIS

WALDO E. SMITH,[25] M. AM. SOC. C. E.[25a]—Many questions that hitherto have been raised concerning the infiltration approach have been answered in this paper, the most notable of which is perhaps the placing of the infiltration curve in proper time position with reference to the rainfall. To the writer's knowledge, this is the most definite work of this sort yet published. The setting down of the step by step procedure clarifies still other questions. A related question that the author himself inspired in a previous paper[26] that yet remains unanswered is the validity of the use of average hourly intensities in the face of the great normal variations in intensity from the hourly average. About one half of the rain within the hour normally falls at two times or more the average hourly intensity, and about one fourth falls at three times or more the average hourly intensity. Thus, the actual pattern of the rainfall that is shown only in terms of hourly averages in Fig. 5 would normally result in decidedly different amounts of rainfall excess. The writer feels, however, that the development of the infiltration technique has been advanced considerably in this paper. An attempt should be made to test this technique as soon as possible, utilizing better basic data.

It is unfortunate that the data for the testing of this method and for this extensive work using the infiltration approach were so inadequate. As the author states, only daily records of precipitation, some of doubtful validity, were available on the watershed for the storms studied. There was no recording rain gage on the area and, for that matter, none was very close to the area until the Garland gage was established in the latter part of the period. Streamflow records consisted generally of twice daily visual observations which, on the watershed of 831 sq.miles, do not define the hydrograph very well. The writer

NOTE.—This paper by W. W. Horner, M. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by L. K. Sherman; September, 1943, by Messrs. Raphael G. Kazmann, and W. G. Hoyt and W. B. Langbein; and October, 1943, by Charlie M. Moore.

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[25a] Received by the Secretary September 22, 1943.

[26] "Surface Runoff Determinations from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1041.

would like to see the author's technique tried with more adequate data, such as those that are available concerning the basin of the Licking River above Zanesville, Ohio. On this basin there are quite a number of recording rain and stream gages with records of 3 yr or 4 yr or longer. These data would afford a much more satisfactory test of the adequacy of this method. Attempting to establish refinements in records, such as hourly values from already uncertain daily amounts of rainfall and such as the sketching in of the gaps between the twice daily stream gaging observations, seems questionable indeed for the uses of the author's analysis.

Inasmuch as the derivation of infiltration curves is by no means standardized and, furthermore, is largely an unknown technique to many hydraulic engineers, a repetition of the citations on the procedure for their derivation presented in the earlier paper,[26] and the addition of more recent ones, is in order. A list, in so far as the writer has it at hand, is appended to this discussion.

The author may have a file of infiltration curves that he has derived for various soil-cover complexes for various seasons, but as yet such curves are not generally available. Derivation is laborious and within the writer's experience there is frequently little uniformity in product as derived by the different methods for the same single watershed complex and storm.

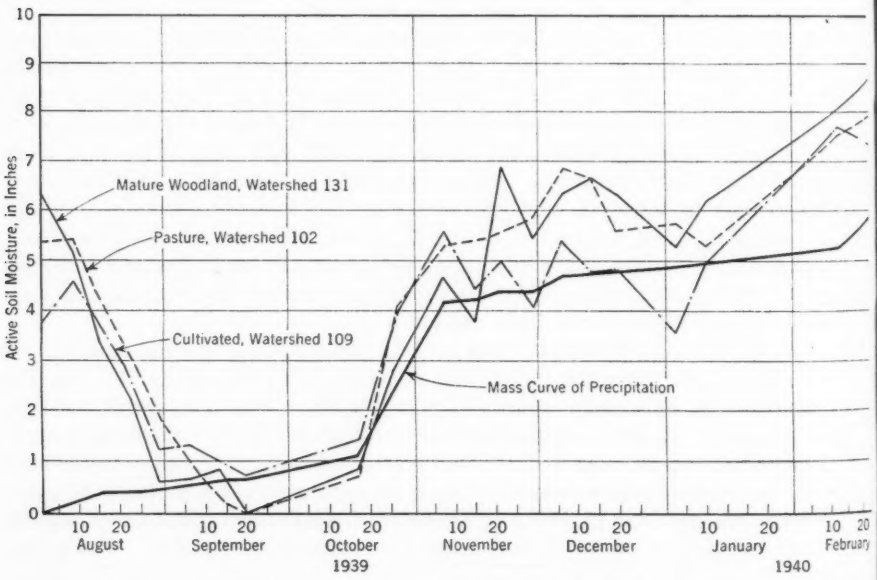
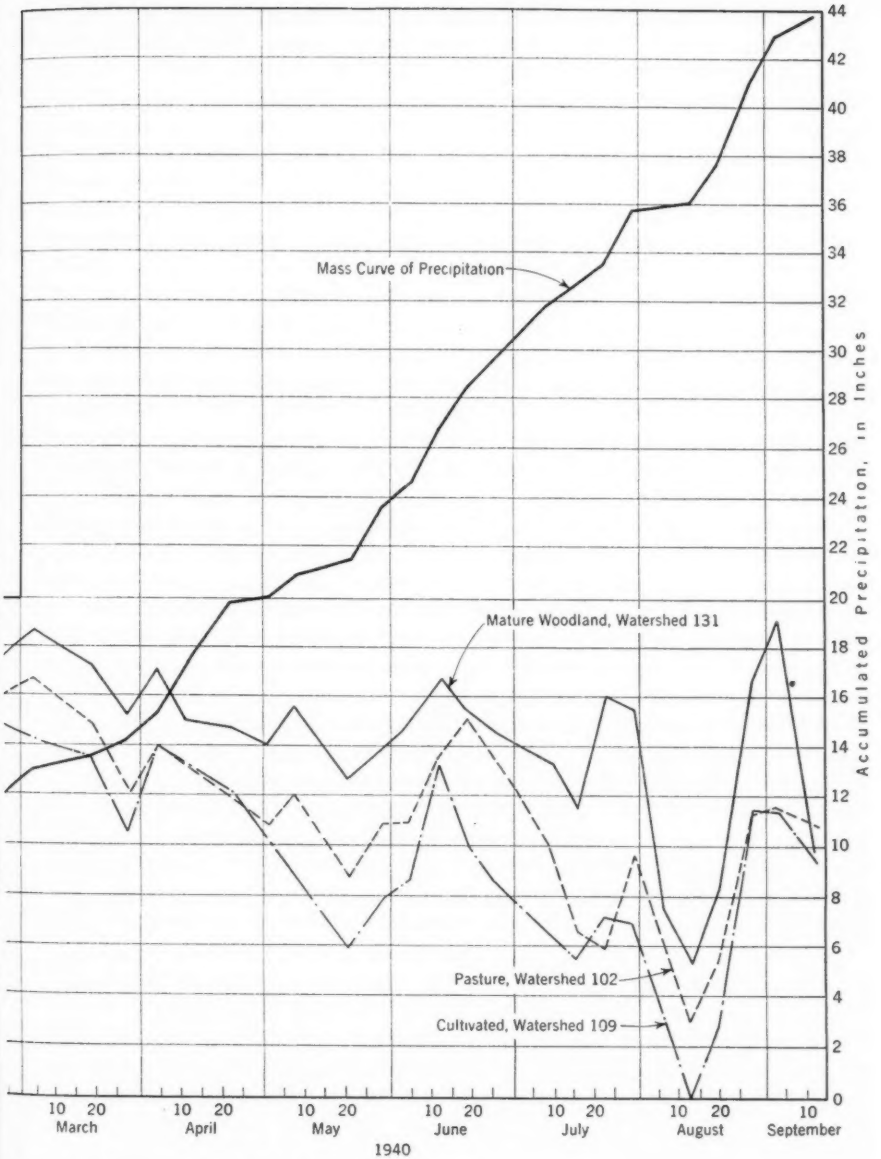


FIG. 9.—CHARACTERISTICS OF "ACTIVE SOIL

With regard to the author's discussion of outstanding flood periods in Part III of this paper and to his statement that floods are usually caused by the coincidence of storms and low infiltration rather than by either of these things



WATER" IN THE UPPER 40 IN. OF SOIL MANTLE

alone, a paper by F. R. Dreibelbis and F. A. Post [27] should be called to the attention of hydraulic engineers. Messrs. Dreibelbis and Post showed that

[27] "An Inventory of Soil-Water Relationships on Woodland, Pasture, and Cultivated Soils," by F. R. Dreibelbis and F. A. Post, *Proceedings*, Soil Science Soc. of America, Vol. 6, 1941, p. 462.

the "active soil water" in the upper 40 in. of the soil mantle that they studied varies as much as 9.39 in. in depth of water in a year. Three curves of the "active soil water" prepared from their data [28] are shown in Fig. 9. On this illustration also is given the mass curve of precipitation. Volume-weight soil samples were taken at intervals of a week to a month on the dates generally shown by the break points in the curves, and the actual quantity of water in each sample was determined. That water greater than the minimum for the year was defined as "active soil water." There was still considerable moisture in the soil at the time of the minimum value, which, in the case of watershed 109 (cultivated), was 6.28 in.[29] of depth. The "active soil water" in the graphs shown is probably considerably more than the subsurface retention (capillary and pellicular water) in the 40-in. layer, and includes subsurface detention (gravitational water). The soil probably approached saturation at the wettest time of the year. Infiltration values are low at such times in these soils and those of the Texas Blacklands. This method may possibly be used as a quantitative indicator of low infiltration rates and approach of the soil to saturation and thus of incipient flood conditions. More detailed information on the vertical distribution of the water in the 40-in. layer might prove helpful.

It is interesting to note that in general the mass curve shows light precipitation during periods of decreasing active soil moisture, and heavy precipitation during periods of increasing active soil moisture. This is a logical deduction supported by the data. Furthermore, due to high evapo-transpiration in the summer and due, perhaps in part, to the usual nature of many summer storms (high intensity and short duration), much more precipitation is necessary to support a constant value of active soil moisture in summer than in winter. Still a further fact is worthy of note—namely, that the greatest depth of active soil moisture occurred on woodland (9.39 in.), next on pasture (8.36 in.), and lowest on cultivated land (7.82 in.). (In the paper cited,[29] this value is given as 7.09 in., but the period used ended August 7, 1940. Extending the period to September 12 gave a new low value for the cultivated watershed, increasing the range between lowest and highest values of soil moisture to 7.82 in.) Data on retention and detention will probably prove to be very useful in analyses made using the infiltration approach.

It is further interesting to note that the active soil moisture fluctuations for the 13-month period conform to the trend of precipitation-evaporation ratios in so far as evaporation data are available for their determination. These ratios, the usefulness of which were established by C. W. Thornthwaite,[30] are simply the monthly precipitation divided by corresponding monthly evaporation. They may also be derived by empirical means from the data on temperature and precipitation. This was not done. The ratios for the available data might have been added to Fig. 9 but were omitted to avoid confusion. For the months for which they can be found they are given below:

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[28] "An Inventory of Soil-Water Relationships on Woodland, Pasture, and Cultivated Soils," by F. R. Dreibelbis and F. A. Post, *Proceedings*, Soil Soc. of America, Vol. 6, 1941, Table 1.

[29] For information concerning the watersheds involved, and for much of the data used by Messrs. Dreibelbis and Post, see "Hydrologic Data," *Hydrologic Bulletin No. 1*, North Appalachian Experimental Watershed, Coshocton, Ohio, 1939, and "Hydrologic Data," *Hydrologic Bulletin No. 4*, North Appalachian Experimental Watershed, Coshocton, Ohio, 1940.

[30] "The Climates of North America According to a New Classification," by C. W. Thornthwaite, *Geographical Review*, Vol. XXI, No. 4, October, 1931, p. 633.

Month	Precipitation-evaporation ratio
1939	
August	0.20
September	0.18
October	0.76
1940	
April	1.34
May	0.71
June	1.34
July	0.57
August	1.44
September	0.42

The ambiguity of the term "infiltration capacity" is indicated in the closing paragraph of the "Synopsis" of the paper in which "infiltration capacity," a rate, is compared with "detention capacity," a volume. As a substitue for the confusing term "infiltration capacity," the writer proposes the term "potential infiltration rate" as being more descriptive of this phenomenon. Furthermore, in the place of "infiltration regression curve," the simple term "infiltration curve" might well be used; or, if one must be more explanatory, the term "the curve of potential infiltration" could be used.

Bibliography of Procedure for Deriving Infiltration Curves.—

(1) **"Analysis of Runoff Plat Experiments with Varying Infiltration Capacity,"** by R. E. Horton, *Transactions*, Am. Geophysical Union, Pt. IV, 1939, p. 693.

(2) **"Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois,"** by W. W. Horner and C. L. Lloyd, *ibid.*, Pt. II, 1940, p. 522.

(3) **"A Graphical Method of Analysis of Sprinkled Plot Hydrographs,"** by A. L. Sharp and H. N. Holtan, *ibid.*, p. 558.

(4) **"Sprinkled Plot-Infiltration and Runoff Experiments on Arizona Desert Soils,"** by E. L. Beutner, R. R. Gaebe, and R. E. Horton, *ibid.*, p. 550.

(5) **"Derivation of Infiltration Capacity from Average Loss Rates,"** by L. K. Sherman, *ibid.*, Pt. I, p. 541.

(6) **"Analysis of Hydrologic Data for Small Watersheds,"** by W. W. Horner, *TP-30*, SCS, Washington, D. C., February, 1940.

(7) **"Infiltration Capacities of Soils in the Macoupin Creek Basin in Illinois,"** by L. K. Sherman, release of the Illinois State Planning Commission, Chicago, Ill., 1940.

(8) **"Determination of Areal Average Infiltration Capacity from Rainfall and Runoff Data,"** by R. E. Horton and Richard Van Vliet, Office of Land Use Coordination, U.S.D.A., 1940.

(9) **"An Approach Toward a Physical Interpretation of Infiltration Capacity,"** by R. E. Horton, *Proceedings* of the Soil Science Soc. of America, Vol. 5, 1940, p. 399.

(10) **"Application of the Infiltration Theory to Engineering Practice,"** by L. K. Sherman and L. C. Mayer, *Transactions*, Am. Geophysical Union, Pt. III, 1941, p. 666.

(11) **"Derivation of Infiltration-Capacity Curve from Infiltrometer Experiments,"** by R. E. Horton, mimeograph release of the Office of Land Use Coordination, U.S.D.A., Washington, D. C., January, 1942, 18 pp.

(12) **"Extension of Graphic Methods of Analysis of Sprinkled Plot Hydrographs to the Analysis of Hydrographs of Control Plots and Small Homogeneous Watersheds,"** by A. L. Sharp and H. N. Holtan, *Transactions*, Am. Geophysical Union, Pt. II, 1942, p. 578.

(13) **"Simplified Method of Determining an Infiltration-Capacity Curve from an Infiltrometer Experiment,"** by R. E. Horton, *ibid.*, p. 570.

(14) **"A Simplified Method of Determining the Constants in the Infiltration-Capacity Equation,"** by R. E. Horton, *ibid.*, p. 575.

P. B. ROWE,[31] ESQ.[31a]—The hydrologic literature concerned with evaluating land effects on stream flow is measurably enhanced by this valuable and stimulating contribution. An approach developed by the California Forest and Range Experiment Station,[32] similar to that presented by Mr. Horner, has been used in the hydrologic analyses of some of the U. S. Department of Agriculture Flood Control Surveys in California. Because of variations in procedures used and in the watershed conditions encountered, a comparison of the two methods and of the results is of interest.

In the Horner method an average (f_a) infiltration-capacity curve of each infiltration "complex" was derived from the rainfall and runoff records of small watersheds. In the California approach, infiltration-capacity curves of the complexes were obtained from small plot (infiltrometer) measurements. These permitted—

(*a*) Reliable determinations, not only of the relative differences of infiltration capacities of the different complexes but also of the range of infiltration capacities within complexes; and

(*b*) Computation of surface runoff from individual complexes by use of the individual *f*-curves or of a series of f_a-curves representing the full range of the infiltration capacities of the complex. The use of such infiltration-capacity curves has proved important in making possible the calculation of surface runoff (often of considerable volume in large complexes) from rainfall at rates below the average infiltration capacity of the complex. Before being used in computing surface runoff the infiltration-capacity curves were adjusted to measured rainfall and runoff of representative sub-drainages comprising single or similar complexes, and the adjusted curves, in turn, were checked against measured rainfall and runoff relations of the watershed as a whole.

In Mr. Horner's analysis, stream-channel inflow was visualized predominantly as surface runoff. After deductions were made for such factors as interception and depression storage, this assumption permitted the direct computation of surface runoff from the infiltration-capacity curves and storm precipitation patterns. In the watersheds studied in California more than 10%

[31] Silviculturist, Div. of Forest Influences, California Forest and Range Experiment Station, maintained by the U. S. Dept. of Agriculture, Forest Service, in cooperation with the Univ. of California, Berkeley.

[31a] Received by the Secretary October 27, 1943.

[32] "A Method of Hydrologic Analysis in Watershed Management," by P. B. Rowe, Twenty-Fourth Annual Meeting, Am. Geophysical Union, April, 1943 (in process of publication).

of the flows at flood crest during major storms, and much higher proportions during minor storms, were composed of "quick subsurface storm flow." Further, under watershed conditions with an assumed improved land-use program in operation, the proportion of channel flow at flood crest composed of quick subsurface storm flow was normally much greater than under present watershed conditions.

TABLE 8.—COMPUTED SURFACE RUNOFF AND FLOOD CREST DISCHARGES FOR PRESENT AND ASSUMED LAND-USE CONDITIONS, SANTA YNEZ RIVER DRAINAGE, CALIFORNIA

Watershed land-use conditions	3.50-IN. STORM		6.00-IN. STORM	
	Runoff[a]	Discharge[b]	Runoff[a]	Discharge[b]
Present observed	1.08	51,600	2.15	105,000
Assumed without program	1.17	55,700	2.27	110,200
Assumed with program	0.89	43,600	1.77	90,000

[a] Surface runoff, in inches of depth. [b] Crest discharge, in cubic feet per second.

In some watersheds considerable amounts of surface runoff were also yielded to channel inflow from complexes of shallow soils that, owing to impervious or partly impervious soil and rock formations or limited water-storage capacities of the substrata, became saturated during the storm. In applying the infiltration-capacity curves to the computation of surface runoff from these complexes, it was necessary to determine not only the rates of the quick subsurface storm flow throughout the course of the storm but also the depletion rates in storage capacities of the soil and substrata of the complexes reaching saturation. This was accomplished largely by analysis of rainfall and runoff records of subwatersheds composed of representative complexes and by establishing correlations between rates of quick subsurface flow and the total amounts of water retained within the soil and substrata.

In both the Horner and the California methods the infiltration-capacity curves were correlated with soil moisture and other antecedent watershed conditions to determine the correct position of the curves with respect to the storm rainfall patterns used in the surface runoff calculations. Mr. Horner wisely stresses the importance of this operation and, for analyses in which storm patterns are fitted to average watershed conditions, presents a very workable method for accomplishing it. Of equal importance in California is the placement and use of the correct infiltration-capacity curves in complexes comprising large areas of recently cultivated or freshly burned-over lands. On such lands the infiltration capacities may be decreased rapidly by changes in the physical properties of the soil through exposure to the beating effects of rain, surface runoff, and soil erosion.

Table 8 presents a comparison of the computed surface runoff and flood crest discharges during mid-winter storms in the Santa Ynez River drainage under present and assumed land-use conditions, with and without a remedial program. The Santa Ynez watershed comprises an area of 790 sq miles in the chaparral

(brush) type of the Coast Range Mountains in Santa Barbara County, California. Owing to the high soil-moisture content and comparatively low rate of water use by vegetation during the winter flood periods, the computed reductions in surface runoff were accompanied by almost equivalent increases in subsurface and ground-water flows. The reduction in flood crest discharges expected from the proposed remedial land-use program results, therefore, not so much from a reduction in total water yield as from the more equable distribution of the yield.

C. S. JARVIS,[33] M. AM. SOC. C. E.[33a]—It is much more satisfactory to peruse the report of a consulting engineer on the results actually achieved and recorded than to scan the prospectus of intended goals in soil and water conservation. Many of the technical papers thus far dealing with the rôle of the land in regulating runoff in general and flood flow in particular have perforce depended on assumptions, estimates, and specifications which may or may not represent the changed conditions after the contracts are approaching completion. The author has carefully presented his official observations and interpretations in such a manner as to convince the most skeptical of the following facts:

1. The East Fork tributary of the Trinity River above Rockwall—not specially favored as to topography, soil, natural vegetative cover, rainfall distribution, and other significant elements of the geologic and climatic background, but somewhat deficient in some of the characteristics that would assist in conserving moisture where it falls—is apparently amenable to the improved land use and management, with a corresponding reduction in peak discharge ranging usually from 10% to 40%.

2. With such beneficial disposal of considerable percentages of the most notable storms of record during the past few years, the conclusion is inescapable that greater percentages of retention and infiltration would occur with the more numerous storms of less severity.

3. Contrary to the conservative forecasts of prominent hydrologists (who claimed to be friendly toward conservation principles, yet disavowed any hope of influencing the catastrophic severity of the flood by the surface treatment and land-use changes included in the conservation program), it appears that measurable reduction can be assured in the event of long-period maxima, with correspondingly greater effect on flood happenings of greater frequency and less intensity.

4. Wherever the geologic and climatic background is more favorably inclined for response to conservation measures, correspondingly greater reduction of flood crests may result from comparable treatment, as estimated for the North Concho River, Texas,[34] according to preliminary studies and the actual behavior of several thousands of acres of newly treated valley lands. After additional months of study involving the use of infiltrometer equipment of various types on many sample areas throughout the valley, the conclusion was reached by the consultants in charge that the preliminary estimates were con-

[33] Cons. Engr., Washington, D. C.

[33a] Received by the Secretary October 28, 1943.

servative as to assumed infiltration capacities and rates that would prevail. The later findings would have supported a more liberal estimate of flood-peak reduction, as well as the decrease of total volume, than that shown under the hydrographs presented by the writer on a previous occasion.[34]

[34] "Analysis of Run-Off Characteristics," by Otto H. Meyer, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 132, Fig. 13.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA

Discussion

BY PAUL V. HODGES, EDGAR E. FOSTER, R. W. DAVENPORT, AND A. K. SHOWALTER

PAUL V. HODGES,[18] M. AM. SOC. C. E.[18a]—The assembly and analysis of rainfall records from the many stations, together with the characteristics of certain heavy storms, add a great amount of valuable data to the somewhat meager information available for this area. In Table 3, the summer rains, as the author states, usually occur in periods of less than 10 hr, and therefore this fact should be considered in the use of the table for flood studies.

The shorter the period of rainfall, the greater the number of rains, probably, in inverse proportion to the rainfall period; whereas summer rains of one or two days would be relatively infrequent.

An example of a rainfall of long duration is the storm producing the San Pedro River flood of September 28, 1926, which had a momentary peak discharge of 98,000 cu ft per sec and a calendar-day discharge of 28,800 cu ft per sec at Fairbank station. The average precipitation over the basin for a 7-day period (September 24 to 30) was 5.04 in., producing a runoff at the Fairbank station of 110,000 acre-ft or 1.38-in. depth over the area during the same period—that is, 27.4% of the precipitation.

From a study of the record flood flows of the upper Gila Basin, a formula representing the relation of momentary peak discharge in cubic feet per second to drainage area in square miles was derived.

The formula $Q = 2{,}070\,A^{0.44}$ represents the maximum recorded floods of the larger rivers, and about the average of flood estimates of many small tributaries. This formula gives the following peak discharge:

NOTE.—This paper by Luna B. Leopold, Jun. Am. Soc. C. E., was published in February, 1943. *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by Lawrence Pratt, Assoc. M. Am. Soc. C. E.; June, 1943, by Messrs. L. L. Harrold and A. J. Dickson, and James Girand; and November, 1943, by Clarence S. Jarvis.

[18] Care, U. S. Engrs. Office, Omaha, Nebr.

[18a] Received by the Secretary October 28, 1943.

A, in square miles	Discharge, in cubic feet per second	Discharge, in inches per hour
1	2,070	3.20
5	4,200	1.30
10	5,700	0.88
20	7,700	0.60
30	9,250	0.48
50	11,600	0.36
100	15,700	0.24

Summer rains are very erratic in size and location, and records at any one station would include but a small percentage of the total number of storms, and those of the highest intensity would very rarely be recorded or measured at any particular station. For this reason the data, for most of the stations, are considered insufficient for determining the maximum rainfall to be expected over long periods of time.

The isopluvial map of New Mexico as developed in Fig. 5 shows higher 24-hr rain than that indicated for western Texas by the probability chart for 50-yr frequency, shown in "Excessive Rainfall in Texas."[19] This is due, no doubt, to the longer record available to the author.

The writer made a study of the rainfall and runoff of the upper Gila River Basin in Arizona and New Mexico (unpublished report of August 14, 1936), in which he determined the average precipitation for different periods of the year, for the precipitation stations having 19 to 38 years of record. It was determined that the average precipitation for the month of July was more than three times the monthly winter precipitation on an average, and that the average precipitation during July, August, and September was greater than that for any other period, being 48% of the yearly total for all stations during periods of record.

The average number of days with a precipitation of 0.01 in. or more was computed for the 15 upper basin stations having a station-year record of 315 years. The distribution and average number of days, for different periods of the year, were found to be:

Period	Average number of days	Percentages
April to June	7.48	13.7
July to September	24.22	44.5
October to March	22.80	41.8
Twelve months	54.50	100.0

From this study it seems that the summer storms have a greater daily precipitation than other storms; that is, the summer storms, representing 44.5% of occurrences, produce 48% of the annual precipitation.

Mr. Leopold has presented much information of great value on intensity and the occurrence of rains at the various stations. Although the data for some of the stations may give reliable indications, the great variation in rainfall

[19] *Bulletin No. 25*, State of Texas, Reclamation Dept., November, 1934, p. 110.

between near-by stations would tend to discount the value of many predictions of maximum rainfall in Table 3. With longer records the maximum rainfall would tend, no doubt, to be more uniformly distributed.

EDGAR E. FOSTER,[20] ASSOC. M. AM. SOC. C. E.[20a]—A fundamentally sound analysis of the precipitation in the two southwestern states, New Mexico and Arizona, has been presented in this interesting paper. The author has extended the analysis to the basic causes, such as type and movements of air masses, type of storms, and effect of topography. These and related climatic factors should be the first items considered in an investigation of the hydrology of a given region, since they are the underlying causes of the occurrence and distribution of precipitation.

The region under consideration is somewhat off the usual paths of air masses affecting the United States other than the adjacent areas of the Great Basin and Colorado Plateau, and it is protected furthermore from invasion of outside air by the border of high mountains. It is isolated meteorologically to the extent that H. C. Willett[21] designates it as a source region of tropical continental air masses. Since air must remain quiescent to acquire the characteristic properties of a source region, the applicability of such a designation indicates that there could be relatively little interchange of air with outside areas.

This meteorological isolation probably promotes the spotted, irregular summer precipitation. This irregularity, added to the effect of the rugged, mountainous topography, would make difficult the construction of isohyetal maps of storms, as the author has indicated by the omission of such maps from his paper. Furthermore, the data of precipitation at one station should be used at other points only after a careful consideration of all pertinent factors that may vitiate the results. For this reason the station-year method of constructing frequency curves would be open to some question because topographical features may cause sufficiently discordant data to offset climatic homogeneity. The author wisely has advised caution in the use of the recurrence-interval data, even through curves were drawn for each of the stations listed.

The writer questions the use of the term "recurrence-interval" when referring to data or curves that show frequency. These are in reality "frequency data" or "frequency curves." It appears desirable to adhere to commonly and widely accepted terms rather than those relatively unknown. The data of 1-day rainfall which probably will be equaled or exceeded once in the specified period of years may be obtained from frequency curves that have been expressed in the integral form as $P = \int_{-\infty}^{+\infty} e^{-u^2}\, du$. The fact that the data were taken from empirical curves would not alter this principle.

Where rainfall frequencies are important factors in an investigation, the writer suggests that they be computed by methods based on the mathematical theory of statistics or probability. For this purpose the unbounded function

[20] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[20a] Received by the Secretary November 6, 1943.

[21] "American Air Mass Properties," by H. C. Willett, Papers in Physical Oceanography and Meteorology, Mass. Inst. of Tech. and Woods Hole Oceanographic Inst., Vol. 2, No. 2, 1933.

proposed by J. J. Slade[22] is recommended. The use of this function involves considerably more work, but it takes into proper consideration the "widely-spaced points in the upper limits" which are the values of rare intense rainfall.

R. W. DAVENPORT,[23] M. AM. SOC. C. E.[23a]—The derivation of Fig. 5 showing, for Arizona and New Mexico, the 24-hr rains equaled or exceeded once in 50 years may well be considered in the light of inherent statistical characteristics of the basic data. These data are summarized in Table 3.

Out of the 283 station records that are used, two records have frequency arrays of 50 years or more. The average length of record is approximately 27 years. Substantially, therefore, the records may be considered as offering 283 samples of the highest 24-hr rain in periods of about 27 years. The extrapolation of the frequency arrays to show rains equaled or exceeded once in 50 years produces results that fall short of being equivalent to actual samples of the highest 24-hr rains experienced in periods of 50 years. For purposes of this discussion, however, the data in the "50 years—all year" column of Table 3 will be considered as representing maximum 24-hr rains in 50-yr records at the respective stations.

If long-time frequency relations for 24-hr rainfalls at the respective stations were available, the 50-yr maxima could be assigned frequencies according to their positions on the magnitude-frequency scales. One maximum might be such as the long-time relation showed would be equaled or exceeded once in 40 years, another once in 75 years, another once in 150 years, etc.

The probability that the value to be equaled or exceeded once in 50 years would be equaled or exceeded in a given year is $\frac{1}{50}$. The probability that such value would be equaled or fallen short of in a given year is $\frac{49}{50}$ and in a 50-yr period $(\frac{49}{50})^{50}$, or 0.364. This signifies that, in a large group of 50-yr samples, the maxima in 36.4% would equal or fall short of the values to be equaled or exceeded once in 50 years.

Similarly the value to be equaled or exceeded once in 75 years would be equaled or fallen short of $(\frac{74}{75})^{50}$, or 51.1%, of the samples. The corresponding results for values to be equaled or exceeded once in 100 years and once in 500 years are 60.5% and 90.5%, respectively.

TABLE 5.—DISTRIBUTION OF 283 MAXIMA ACCORDING TO PERCENTAGES

Frequency scale[a] (1)	% (2)	Distribution (3)
50 years or less.....	36.4	103
50 to 75 years.....	14.7	41
75 to 100 years.....	9.4	27
100 to 500 years.....	30.0	85
500 years and more....	9.5	27

[a] Equaled or exceeded once in the number of years indicated.

The foregoing shows that for a large number of 50-yr samples the maxima would be distributed on the frequency scale according to the percentages shown in the following tabulation. If the distribution of the 283 assumed maxima of Table 3 conformed to these percentages, the classification of the station records according to frequencies would be as shown in Col. 3, Table 5. Of course, it

[22] "An Asymmetric Probability Function," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 35.

[23] Chf., Div. of Water Utilization, Dept. of the Interior, U. S. Geological Survey, Washington, D. C.

[23a] Received by the Secretary November 8, 1943.

would be possible to have a great variety of other distributions in samples limited in number to 283, but that shown is reasonably typical.

If frequencies were assigned to the values shown in Figs. 3 and 4, they would be distributed somewhat as shown in Col. 3, Table 5. The median frequency would be that equaled or exceeded once in slightly less than 75 years.

It would seem that, in the definition of isopluvial lines by such data, considerable uncertainty would be involved as to the extent to which they represent variations of magnitude incident to topography and other local causes or variations incident to the use of samples of limited size. The latter variations might tend to obscure the former. Moreover, the isopluvials developed might correspond most closely to the median of the frequencies represented; that is, 24-hr rains equaled or exceeded once in 75 years. Viewed in the light of the uncertainties discussed herein, the significance of Fig. 5 is not clear.

The author is to be commended for his extensive compilation of rainfall data and the presentation of the results in compressed form convenient for reference and use.

A. K. SHOWALTER,[24] ESQ.[24a]—This paper is a contribution to the hydroclimatic history of New Mexico and Arizona. The results are representative of the available data, but it is doubtful whether point-rainfall frequencies can indicate directly the frequencies of selected average depths over specific drainage areas. The author demonstrates, for example, that rainfall is more frequent at higher altitudes but that, nevertheless, in a single storm the heaviest rainfall usually occurs near the base of the mountain. If data on frequency of average depths of rainfall over a small, steep drainage area were desired, the results would differ radically if in one case the key station were leeward of a high mountain and in another case at a low elevation directly exposed to the prevailing storm wind.

If Mr. Leopold's data are to be used in design, it is recommended that pilot studies be conducted to determine the typical relationship between the depth of rainfall at the key station and the average depth over the project basin. Such studies would seem to be especially important in regions dominated by pronounced orographic influences.

More complete data are needed on typical depth-area-time relationships in this region, and it is probable that adequate data will never be obtained without the use of very dense networks of electrically synchronized automatic rain gages. Sampling networks should be located in the relatively flat areas and also in regions of marked topographic variation.

The author neglected to mention that tropical Pacific hurricanes sometimes move up the west coast of Mexico or through the Gulf of California bringing with them deep currents of moist air and subsequently heavy widespread rains over parts of Arizona or New Mexico long after surface barometric indications of the tropical cyclone have disappeared.

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[24] Meteorologist in Charge, Hydrometeorological Section, Weather Bureau, U. S. Dept. of Commerce, Washington, D. C.

[24a] Received by the Secretary November 16, 1943.

Another storm type not mentioned is a deep, cold, cyclonic circulation which may move slowly across the region and cause the northward-flowing, moist, tropical, maritime air to the east to curve cyclonically as it advances up slope through Texas and northern Mexico into New Mexico and Arizona. Such a circulation pattern in a surface air mass must necessarily be accompanied by horizontal convergence and vertical stretching which tend to accelerate the release of potential instability and convective overturning. Such patterns are typical of eastern Colorado and New Mexico from late April through early June.

Mr. Leopold's data and conclusion regarding the relationship between rainfall intensity and height seem more consistent with theoretical deductions than much of the previous literature on this subject. The opportunity for rainfall is increased at high-level stations by the increased lift, and therefore there is a higher frequency at the higher elevations if only average rains or rains of low magnitude are considered. However, as the height increases, the depth of available precipitable water in the air above decreases (neglecting localized chimney effects which carry suspended condensation particles), and also the zones of horizontal convergence are not so well defined. The result is lesser frequency of excessive rains of a given magnitude at the higher elevations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE A SYMPOSIUM

Discussion

BY EGIDIO O. DI GENOVA, AND CHARLES MACKINTOSH

EGIDIO O. DI GENOVA,[21] Assoc. M. AM. SOC. C. E.[21a]—An assertion, repeated several times in this Symposium, implies that the elastic theory is not entirely valid for the Rainbow Arch Bridge. The writer's extended studies of the elastic theory lead him to the conviction that the following statement in the first paper (12 lines preceding Eq. 1) at least is out of place:

> "The preliminary calculations for the two-hinged arch rib were sufficient to show that the elastic theory was inadequate for the design of such a structure."

If the writer understands this statement correctly, he believes that it is incorrect. The two-hinged arch is different from the hingeless arch, and it is impossible to draw parallel comparisons between the two. In general, in any elastic system the writer has found that the resultant elastic deformations (and therefore the internal stresses), computed by means of the so-called elastic theory, and the findings obtained by other systems of computations are closely alike, with one exception: When all the possible internal stresses are considered (that is, compression, shear, and flexural moments, generated by an externally applied system of forces), slightly larger results are obtained; but, if the structure is computed by the elastic method without taking in consideration shear and axial stresses, the difference does not exist, or it is of a very small relative order such as can be expected within the common range of approximations.

The elastic theory in itself has certain limitations in relation to the magnitude of the elastic deformations, but those limitations must be considered in relation to the linear variation of strain and stress of the material used, and also in relation to the relative size of the structure.

NOTE.—This Symposium was published in October, 1943, *Proceedings*.

[21] Civ. Engr., Board of Water Supply, New York, N. Y.

[21a] Received by the Secretary October 21, 1943.

On an arch of 950-ft span and 150-ft rise, 12 in. (or even 18 in.) of elastic deformation, for a steel structure, is relatively small, and the limits imposed by the theory of elasticity are still satisfied.

The writer has the impression that the geometrical simplicity of this arch, the continuous uniformity of its cross section, the very sturdy condition of fixed restraint, etc., define an ideal case for useful practical application of the theory of elasticity.

Although this theory is not applied to any minor engineering problem, nevertheless the use of it by well-trained engineers in this field and for important works is worth a lot. All engineers know that, in theoretical studies inherent in the elastic behavior of materials, the elastic theory is a very important instrument for developing ideas, but there are also many cases in which its application to practical problems is just as important as the more theoretical applications and the Rainbow Bridge, in the writer's opinion, is one of them.

This writer insists on the idea that the basic principles governing the elastic theory must be understood clearly, and a consistent system of computation must be followed implicitly. Tables and methods of application must be used wisely so as to reflect the theory accurately, or there is likely to be considerable confusion in the mind of the computer; if this advice is not followed, the chance of errors is greater, especially if certain sets of formulas are applied blindly.

The writer admits that it is very difficult to establish a logical system of computations for such an important structure as the Rainbow Bridge. Nevertheless, the various means of checking one's work offered by the elastic theory (as for example: The cut-and-try method; the method of virtual work; the ellipse-of-elasticity method; etc.), which are integral parts of that theory, can be used to best advantage on a project that is as important as the Rainbow Bridge. The results obtained are certainly worth the extra cost, and no competent person can ever charge that they are erroneous, unless some serious error is made.

After a preliminary proportioning of the structural elements, the use of the various analytical and graphic solutions offered by the elastic theory is a blessing. Even if the expense of constructing a model is considered desirable, the theory itself will prove valuable.

The elastic theory, like many other means of analysis, is not magic; the engineer should always remember that in final results he receives from an analysis only the findings inherent in the assumptions made. For example, in the design of the Rainbow Bridge, the effect of vertical loads and the deflections of the arch ribs could have been analyzed by the elastic theory, whereas the laterals, the deck, and the spandrel columns could have been analyzed in some other way. In the preliminary analysis of the bridge, it seems natural that the spandrel columns, for example, should be replaced by their reactions in order to simplify the computation of the arch itself.

The application of the elastic theory requires a little more than just a handbook. It calls for personal ability and engineering art—qualities that cannot be taught easily by any standard handbook.

It is also true that, after all, the origin of any system of computations is essentially based on the elastic theory. At times some standardized system of

analysis, made easy for a practical man, is a wonderful thing indeed, for the diffusion of knowledge; but, as a class, professional men of higher standards and reputation must remember that the basic culture of an engineer must be on a plane above the superficial knowledge of a plain mechanic, in the same proportion as the skill of the physician is to that of the registered nurse. To this end, the writer takes this opportunity to express the hope that one more year may be added to the present university engineering course so as to spur the profession to attain a keener technical, cultural, and moral point of view and that engineers of the future will use it to sponsor a more extensive and rational study of the elastic theory.

CHARLES MACKINTOSH,[22] ESQ.[22a]—This analysis of the Rainbow Arch Bridge is a valuable work because the writers have reemphasized the fact that any slender arch must be considered as an unstable body since twice the load produces more than twice the stresses. An attempt has been made to reduce the cumbersome procedure required for finding the final moments in the arch. In developing an approximate method of finding the moment, a formula is suggested which is quite satisfactory for the particular arch under consideration with the particular loads to which it was subject. However, these recommended formulas are not applicable to many other loads and shapes of arches.

If the arch were of such shape that the moment due to the dead load was high, Eq. 16 would not be satisfactory. According to this formula, for instance, if there were no live loads or impact loads, the arch moments would not be increased by the slenderness of the arch section. Perhaps Eq. 16 would be satisfactory for the general case if all of the moments were grouped together ($M_{DL,\,LL,\,I,\,w,\,t}$) and these moments, in turn, were multiplied by a factor using the combined moments in the denominator similar to Eq. 16, as previously written.

Thus

$$s_y = N\frac{\Sigma T}{A} + N\frac{\Sigma M}{S}\left[\frac{1}{1 - N\,\Sigma T\,\dfrac{\Sigma u}{\Sigma M}}\right] \quad \ldots\ldots\ldots\ldots (35a)$$

To include the effect of the inaccuracy in fabricating the arch to the prescribed outline, ΣM should include $\Sigma T \times e'$, in which e' is the tolerance allowed as the authors have indicated in Eq. 17. If no factor of ignorance is present in determining e', the ratio $\dfrac{e'}{N}$ may be used. Eq. 21 would also be unsatisfactory in any case where M_{DL} is large. This may be treated as suggested herein.

In the course of designing many long-span slender roof arches, the writer has found that a rapid and convenient approximation is given by an adaptation of

[22] Structural Engr. (Mackintosh & Mackintosh), Los Angeles, Calif.

[22a] Received by the Secretary November 20, 1943.

the usual beam-column formulas[23] as follows:

$$s_y = N\frac{\Sigma T}{A} + N\frac{\Sigma M}{S}\left[\frac{1}{1 - N\dfrac{\Sigma T}{\Sigma T_{cr}}}\right] \quad \ldots\ldots\ldots\ldots (35b)$$

in which

$$T_{cr} = \frac{\pi^2 E I}{L^2_i}\left[1 - \left(\frac{\alpha}{\pi}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

In Eq. 36, L_i = estimated length of arch between points of inflection, and α = central angle between points of inflection. For flat arches the correction for curvature $\left[1 - \left(\frac{\alpha}{\pi}\right)^2\right]$ is small and may be neglected.

Such methods offer a facile check for stresses in "lamella roofs" and in arched roofs with ribs of wood, concrete, or steel under uniform loads and one-side snow loads and wind loads.

[23] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 27–45 and 225–230; "Formulas for Stress and Strain," by Raymond J. Roark, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1938, pp. 120–121; "Strength of Materials," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, 1940, Vol. 2, p. 218; and "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, 1938, Chapter 14.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

BY HUGH STEVENS BELL, AND HARRY F. BLANEY

HUGH STEVENS BELL,[10] ESQ.[10a]—Under the heading, "The Action of Sediment in Reservoirs: Density Flows in Lake Mead," Mr. Witzig attributes to the writer (25)[10b] an estimate that "silt-laden density flows in Lake Mead carry about 24% of all sediment deposited in the lake." For the sake of accuracy it seems desirable to quote directly from the passage cited by Mr. Witzig (25), omitting any parts not pertinent to the present discussion:

> "Density currents * * * were transporting fine sediments into lower Lake Mead at the rate of 875,000 tons per week during the 61 months immediately following the closing of the (diversion) tunnel on May 1, 1936. * * * If it is assumed that these sediments will eventually compact to 60 pounds per cubic foot, the density currents are transporting material sufficient to occupy 1 percent of the original spillway-crest capacity each 8.2 years.* * *
>
> "To see what this might mean in terms of the total life of Lake Mead, assume that, under present conditions, the reservoir will become completely filled with sediments in 200 years. According to the estimates above, material transported by density currents will occupy approximately 24 percent of the original storage space at that time, provided that none is discharged, and the average concentration has become 60 pounds per cubic foot."

The 24% figure, then, is based upon two assumptions—(1) that density current deposits will contain 60 lb per cu ft of sediment and (2) that Lake Mead will be completely silted up in 200 years. These assumptions were not estimates based upon ample data. They were values chosen because they were convenient and conservative. Therefore, the 24% should not be used, as Mr. Witzig has used it, as a measure of the sediment carried by density currents in terms of the total sediment load brought to Lake Mead. The perti-

NOTE.—This paper by Berard J. Witzig, Jun. Am. Soc. C. E., was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Joe W. Johnson; October, 1943, by Messrs. John W. Stanley, Stafford C. Happ, and Thomas H. Means; and November, 1943, by Carl B. Brown, and C. S. Jarvis.

[10] Associate Soil Conservationist, U. S. Dept. of Agriculture, SCS, California Inst. of Technology, Pasadena, Calif.

[10a] Received by the Secretary October 30, 1943.

[10b] Numerals in parentheses, thus: (25), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

nent value is 875,000 tons per week, or, roughly, 45,000,000 tons per year. In 1928 Messrs. Fortier and Blaney estimated that a reservoir at Boulder Canyon would receive 253,628,000 tons of sediment per year (9). On this widely accepted estimate, the 45,000,000 tons transported annually by density currents would represent, roughly, 18% of the total.

Actually, there is no reason to expect that the fine sediment in lower Lake Mead will attain a mean specific weight that even approaches 60 lb per cu ft during the lifetime of the Boulder Dam project. The several hundred samples taken by the Bureau of Reclamation indicate that a value of 30 to 35 lb is a much more probable one (54).

Before the importance of density currents in the sedimentation of Lake Mead can be evaluated, a reasonable estimate must be had for the mean specific weight of all sediment deposited there. In 1930 Mr. Blaney stated that differences in opinion on this point had resulted in estimates ranging from 80,000 acre-ft to more than 250,000 acre-ft for the average annual volume of sediment that would be deposited. Although a value of 85 lb per cu ft had gained wide acceptance in engineering circles, Mr. Blaney expressed the opinion that 62.5 lb would prove to be approximately correct (55).

At 85 lb per cu ft the estimated annual sediment load of 253,628,000 tons would occupy 137,000 acre-ft. At 30 lb per cu ft, the 45,000,000 tons of this material that is carried into the lower part of the reservoir by density currents will occupy about 70,000 acre-ft, or more than half of the total. Therefore, the sediments not carried by density currents would occupy only 67,000 acre-ft, and would have a specific weight of 143 lb per cu ft. This is an absurd value.

At 62.5 lb per cu ft the total sediment load would occupy about 186,000 acre-ft, of which the 70,000 acre-ft of density current deposits would constitute 37.6%. Sediments not carried by density currents would occupy the remaining 116,000 acre-ft, and would have a mean specific weight of slightly less than 83 lb per cu ft. Although the future may show that even this value is somewhat high, it seems to be a reasonable one.

It may be estimated with some confidence, then, that density currents are transporting not 24% of all sediment at Lake Mead, but from 35% to 40% of it in terms of the volume after deposition.

Not only do density currents transport those sediments that form deposits with specific weights that are very low, but also, by sorting and carrying away the fine particles that would fill a part of the interstices in the sediments left behind, they decrease the specific weight of even the coarser deposits. The economic importance of density currents, therefore, is tremendous. If it were possible to operate the reservoir so that 50% of the sediment carried by density currents could be vented, rather than stored, the life of the project would be increased by approximately 25%. The seemingly insurmountable obstacles that prevent this from being accomplished are human, not physical ones.

Mr. Witzig states (heading, "The Action of Sediment in Reservoirs: Formation and Dispersion"), "The loss of energy as the stream enters the reservoir is not generally sufficient to prevent the formation of a density current." This statement is correct, but may give a false idea as to the type of energy that produces density currents. It is kinetic energy that a river loses or releases

upon entering a reservoir. Since this type of energy causes mixing between the river water and the reservoir water, the more of it there is available the less chance there is for a river to become a density current. If there is enough kinetic energy to make a homogeneous mixture of the river water and all the water stored in the reservoir, there can be no density current because there will not be two adjacent fluid masses of approximately equal density. On the other hand, if the kinetic energy is not sufficient to produce such a mixture, the river water may still possess great potential energy because, although it has been diluted by entrance mixing, it has a density that differs from that of the unmixed reservoir water. It is this potential energy, rather than any residual kinetic energy, that makes density currents possible.

HARRY F. BLANEY,[11] M. AM. SOC. C. E.[11a]—For many years engineers have recognized the need for more definite information concerning the silting of reservoirs under varying conditions. The author has raised this question again and his paper indicates the need for further research in the prevention of sedimentation in reservoirs. The writer believes, however, since determinations of silt content of river water usually are made on a weight basis, that the presentation of more data on the relation of volume and weight of silt would add to the value of the paper. Such information is needed to convert the results of silt samplings on a weight basis to volume, as the engineer is more interested in the volume of sediment than its weight.

Under "Origin and Nature of Silt: Definition of Silt," the author states that Mr. Fortier and the writer "recommend for the Lower Colorado a specific weight of 62.5 lb per cu ft for suspended silt, and 85 lb per cu ft for deposits in reservoirs." These estimates were made in 1926. Since then additional information has been collected by various agencies. In 1929 the writer called attention to the fact that differences in opinion as to the weight of 1 cu ft of silt have resulted in estimates of the average volume of suspended silt that would be deposited annually in reservoirs on the Colorado River ranging from 80,000 to more than 250,000 acre-ft per yr (55). At that time the writer was of the opinion that the specific weight of sediment deposited in the bottom of a reservoir created by a high dam at Boulder Canyon would be approximately 62.5 lb per cu ft, provided the deposits were not given an opportunity to dry out and thus destroy their colloidal condition.

The quantity of material deposited in a reservoir that will be affected by alternate drying and wetting is difficult to estimate. When the water level of a reservoir is lowered, material deposited in the upper part of the reservoir will be uncovered, and, upon drying, undoubtedly will shrink in volume. Mr. Faris reported (22) that, in Texas, material deposited under reservoir conditions, and remaining submerged, contained 30 lb or less of dry materials per cubic foot of deposit, whereas material similarly deposited and then subjected to shrinkage through exposure contained in excess of 80 lb of dry material per cubic foot of deposit. These silt load studies have been continued by Dean W. Bloodgood

[11] Senior Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.
[11a] Received by the Secretary November 22, 1943.

of the Division of Irrigation, SCS, in cooperation with the Texas Board of Water Engineers. In a progress report (56) Mr. Bloodgood states that

> "* * * the average weight of the dry material in silt deposits which are continuously submerged approaches 30 pounds per cubic foot. In those deposits which are occasionally exposed, the average dry weight approaches 70 pounds per cubic foot. In deposits where reservoirs are used exclusively for flood control, the average weight ultimately approaches 90 pounds per cubic foot. After a careful consideration of the volume-weight ratios of silt samples in different degrees of consolidation together with the fact that an indeterminable volume of vegetable matter in the form of logs and brush deposited in reservoirs becomes water-logged and lasts indefinitely, seventy (70) pounds was selected as the average ultimate weight of the dry material per cubic foot of deposit in reservoirs where the deposits are subjected to alternate wetting and drying."

The studies of stratified flow in reservoirs by Mr. Bell, mentioned by the author (25), illustrate the need for further research on the sedimentation in reservoirs. Investigations of density flows may provide the means of prolonging the life of large reservoirs.

Since the author's paper was published, the SCS has issued a new publication (52), in which all the known methods of silting control, as described in a rather voluminous but widely scattered literature, and as recorded in the sedimentation studies of the SCS, have been analyzed and classified. This bulletin includes references to some 138 reports. A review of this publication is recommended to those interested in information on methods of controlling or reducing excessive silting in impounding reservoirs.

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(9) **"Silt in the Colorado River and Its Relation to Irrigation,"** by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., 1928.

(22) **"The Silt Load of Texas Streams,"** by Orville A. Faris, *Technical Bulletin No. 382*, U.S.D.A., 1933.

(25) **"Stratified Flow in Reservoirs and Its Use in Prevention of Silting,"** by Hugh Stevens Bell, *Miscellaneous Publication No. 491*, U.S.D.A., September, 1942.

(52) **"The Control of Reservoir Silting,"** by Carl B. Brown, *Miscellaneous Publication No. 521*, U.S.D.A., 1943, 166 pp.

(54) **"Lake Mead Density Current Investigations 1937–1940,"** Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., October 1, 1941, Vol. I, pp. 135–282; also unpublished Bureau of Reclamation data on samples taken monthly from September, 1940, to September, 1943.

(55) *Transactions*, Am. Soc C. E., Vol. 94 (1930), p. 1139.

(56) **"The Silt Load of Texas Streams,"** by Dean W. Bloodgood, mimeographed progress rept., Texas Board of Water Engrs. and U.S.D.A., September, 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRENGTH OF SLENDER BEAMS

Discussion

BY J. P. WADLECK

J. P. WADLECK,[6] ESQ.[6a]—The author should be complimented for his ability to treat the entire scope of his subject with such admirable simplicity. The significance of his paper lies in the fact that, in so far as the writer is aware, this is the first direct attempt to rationalize the problem of lateral stability of ideal beams; that is, to find a mathematical solution to the lateral stability and the corresponding critical stress of real beams with inherent imperfections, eccentricity of load and reactions, possible slight obliquity of beam axis, etc.

The author's paralleling of the knowledge and design practice of the engineering profession on lateral stability of beams, as it is at present, with the situation existing in column design before the introduction of the secant formula, if it were true, would indeed be very fortunate.

The author undoubtedly had in mind knowledge as represented by engineering scientists and students of advanced engineering literature. The average graduate engineer, however, who, long before the existence of any of the "real column" solutions, did have or should have had a clearer understanding of the determining factors, governing stability of struts through Euler's basic column theory, finds that the mechanism of lateral beam failure and its stability theory is still beyond his grasp.

The fact that, thus far, relatively few cases of major failures, which are traceable to lateral buckling of beams, are on record is mainly due to the fact that all rolled sections were safely covered by permissible load versus span tables originating from the AISC handbook (2).[6b] It seems that, no matter how indiscriminating these tables are relative to the degree of end restraint and to type and position of loading, they apparently cover all errors with a margin on the safe side.

NOTE.—This paper by George Winter, Esq., was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. T. McLean Jasper, and C. M. Goodrich.

[6] Development Engr., Niagara Machine & Tool Works, Buffalo, N. Y.

[6a] Received by the Secretary October 29, 1943.

[6b] Numerals in parentheses, thus: (2), refer to the corresponding items in the Bibliography (Appendix I of the paper), and at the end of discussion in this issue.

In the design of long, built-up I-section girders, which otherwise were not intended to resist external lateral loads, to the extent of the writer's knowledge, the designer was and still is aided only by his sense of proportion and conservative, occasionally feeble, rules of thumb, available in various specifications. How such elementary considerations proved adequate to guard against lateral failures of these built-up girders—not covered by the AISC tables—the writer can explain only with the substantial combined lateral bendings and torsional strength of the sections.

As new trends are developed, both in materials and fabricating methods, most of which will result in greatly decreased torsional rigidity of sections (due to the increasing use of welding, to the adoption of high-tensile, low-alloy steels, to the various light-weight and thin-wall formed structures in general), the problem of lateral stability of beams will rise to first-degree importance.

To meet this situation without an extensive research and test program is unthinkable. The existing meager data such as those in the AISC and the other steel handbooks are unusable, due to their limited scope.

To attain the widest validity of test results, with the least effort, it is essential that, in any test program of the future, the work be guided and the results be checked against a rigorous and rational analytical expression. In the establishment of such an approach lies the value of the author's work.

Whether the exact analytical expressions are not too involved for routine design work is another matter. If so, for the purpose of specification writing and tables, they could easily be supplemented with various empirical formulas each within the scope of the respective specification, and the use of the original analytical formula, which is in conformity with the entire range of tests, could be restricted to the treatment of special cases.

Concerning the mathematical phase of the paper the writer calls attention to some interesting possibilities[6c] by transforming Eq. 20, 22, or 23. Substituting the value of A' from Eq. 19 into Eq. 22:

$$\bar{s} = \frac{s_y}{1 + k \sec\left(\frac{I_{x'}}{h}\, \bar{s}\, \frac{L}{\sqrt{D\,C}}\right)} \qquad (73a)$$

in which $D = E\,I_{y'}$ = the lateral flexural rigidity; and $C = K\,G$ = the torsional rigidity of the beam. Recalling that $\frac{1}{2} \times \frac{2\,I_{x'}}{h} \times \bar{s} = \frac{1}{2}\,M$ and $\frac{L}{\sqrt{D\,C}} = \frac{\pi}{M_c}$, thus:

$$\bar{s} = \frac{s_y}{1 + k \sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)} \qquad (73b)$$

or multiplying both sides by $\frac{2\,I_{x'}}{h}$:

$$M = \frac{M_y}{1 + k \sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)} \qquad (74)$$

[6c] Correction for *Transactions:* In the denominator of Eq. 23 insert "sec" after "k."

Inasmuch as values of $\sec\left(\frac{\pi}{2}\times\frac{M}{M_c}\right)$ vary between 0 and ∞ (when $M = 0$ and $M = M_c$, respectively), it would be well worth while to explore the possibility of expressing $\sec\left(\frac{\pi}{2}\times\frac{M}{M_c}\right)$ by a closely conforming rational function, by which Eq. 3 could be transformed into a quadratic equation.

An extremely close approximation to all values of $\sec\alpha$ between 0 and $\frac{\pi}{2}$ (which unfortunately, however, yields a cubic equation) is due to H. A. Webb:

$$\sec\alpha = \frac{1+\frac{4\,\alpha^2}{\pi^2}\times 0.26}{1-\frac{4\,\alpha^2}{\pi^2}} \qquad (75)$$

by which:

$$\sec\left(\frac{\pi}{2}\times\frac{M}{M_c}\right) = \frac{M^2{}_c + 0.26\,M^2}{M^2{}_c - M^2} \qquad (76)$$

If substituted into Eq. 74, Eq. 76 will lead to the expression:

$$(1 - 0.26\,k)\,M^3 - (1+k)\,M\,M^2{}_c - M^2\,M_y + M^2{}_c\,M_y = 0 \qquad (77)$$

By letting $(1 - 0.26\,k) = \alpha$ and $(1+k) = \beta$, and as, in general:

$$M = s\,\frac{2\,I_{x'}}{h} \qquad (78)$$

Eq. 77 can be written as:

$$\alpha\,s^3 - \beta\,s\,s^2{}_c - s^2\,s_y + s_y\,s^2{}_c = 0 \qquad (79)$$

which is solvable for s by either the exact or one of the approximate methods.

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(2) **"Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings,"** Am. Inst. of Steel Construction.

PROCEEDINGS

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(INSTITUTED 1852)

ABRIDGED TRANSLATIONS
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SERIES 2

September, 1943

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

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Series 2

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NOTE.—These translations supplement the nine published in *Proceedings* for November, 1937. They are not subject to discussion, and will not be published in *Transactions*.

FOREWORD

To make available to American engineers important information in the hydraulic engineering field published in foreign languages, a selected group of nine papers was translated, abstracted, and published in *Proceedings* for November, 1937.

The project was set in motion by recipients of the John R. Freeman Traveling Scholarships and carried out under the Society's Special Committee on Hydraulic Research with the cooperation of other interested engineers.

The present group, designated Series 2, was selected with care from a longer list of excellent papers, such as the following:

(1) **"Die Beseitungen von Schwingungen an über-und unterströmten Wehren"** (The Elimination of Vibrations in Overflow and Sluiceway Dams) by Hans Fischer, *Mitteilungen aus den Forschungsanstalten, Gutehoffnunghütte Aktienverein für Bergbau und Nürnberg, Hüttenbetrieb*, Konzern, Berlin, Vol. 5, No. 4, May, 1937, p. 75.

(2) **"Die Schwingungen an unterströmten Wehr und ihrer Beseitungen"** (The Vibrations of Sluiceway Dams and Their Prevention), by Hans Fischer, *ibid.*, No. 5, 1937.

(3) **"Über den Strömungsvorgan an einer unterströmten scharfkantigen Planschütze"** (On the Flow Characteristics of an Underflow Sharp-Edged Sluice Gate), by G. Pajer, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 17, 1937, p. 259.

(4) **"Solution Rigoureuse du Probleme du Clapotis"** (Rigorous Solution of the Problem of Impact of Waves), by M. Larras, *Annales des Ponts et Chaussées*, Vol. 107, September, 1937, p. 392.

(5) **"Strömung in Kanälen mit gekrümmter Sohle"** (Flow in Channels with Vertically Curved Beds), by H. Lauffer, *Wasserkraft und Wasserwirtschaft*, Vol. 31, October 1, 1936, pp. 249 and 260.

(6) **"Das Wasserstrahl und seine Auflösung in Tropfew"** (The Water Jet and Its Disintegration into Drops), by T. Oehler, *Technische Mechanik und Thermodynamik*, Vol. 1, 1930, p. 329.

(7) **"Contributo alla Teoria delle Piene nelle Reti Idrauliche"** (Contribution to the Theory of Floods in Hydraulic Networks), by E. Campini, *L'Energia Elettrica*, Vol. 7, 1930, p. 580.

(8) **"Experience sur les piles de barrage-deversoirs à profile aerodynamique et à contractions milles"** (Experiments on Spillway Piers Having Aerodynamic Profiles and Many Contractions), by L. Escande and G. Sabathe, *Comptes Rendus*, April 6, 1936, p. 1254.

(9) **"Experiments on the Steady Flow in Canals with Lateral Inflow and Outflow,"** by H. Favre and F. Braendle, *Bulletin Technique de la Suisse Romande*, Vol. 63, 1937, pp. 94, 109, and 129.

(10) **"Theoretische Betrachtungen zum Geschiebebetrieb"** (Theoretical Considerations on the Transportation of Detritus), by K. H. Grossman *Schweizerische Bauzeitung*, Vol. 110, 1937, p. 170.

(11) **"Der Mischungsvargan bei platzlichem Querschnittsübergang"** (The Mixing Phenomenon at Sudden Changes in the Cross Section), by C. Jäger, *Wasserkraft und Wasserwirtschaft*, Vol. 24, 1936, p. 306.

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(14) **"Über die Bemessung von Streichwehren"** (On the Dimensions of Side Channel Spillways), by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, Vol. 32, 1937, p. 209.

(15) **"Überströmten, Überfall und Ausfluss"** (Overflow, Overfall, and Outflow), and **"Das Wasserauflaufen"** (Outlets), by F. Bundschu, *Der Bauingenieur*, Vol. 9, 1928, p. 27.

(16) **"Esperienze sul moto perturbato nei canali industriali"** (Experiments on Waves [of Translation] in Industrial Canals), by C. Drioli, *L'Energia Elettrica*, Vol. 14, 1937, pp. 285 and 382.

(17) **"Sul moto delle balle d'aria nelle condotte d'acqua"** (On the Motion of Air Bubbles in Water Conduits), by A. Veronese, *ibid.*, p. 759.

(18) **"Wasserbewegung in steilen Rinnen mit besonderer Berücksichtigung der Selbstbelüftung"** (Flow in Steep Chutes with Special Reference to Self-Aeration), by R. Ehrenberger, *Österreichischer Ingenieur-und Architektenvereines*, Nos. 15/16 and 17/18, 1926, p. 155.

(19) **"Umlenkung eines freien Flussigkeitsstrahles an einer ebenen Platte"** (The Deflection of a Free Fluid Jet by a Flat Plate), by W. Schach, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 80, 1936, p. 57.

(20) **"Berechnung der Verluste im Verbindungsteil zwischen einer breiten Entnahmestelle und einem schrägen Kanal"** (Calculation of the Losses at the Transition from a Broad Intake to a Transverse Canal), by D. Sokolow, *Wasserkraft und Wasserwirtschaft*, February 16, 1936, p. 43.

(21) **"Zum Zerfall eines Flussigkeitsstrahles"** (The Disintegration of a Fluid Jet), by C. Weber, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 11, 1931, p. 136.

(22) **"Investigation of Wave Motion in Long Pools"** (in Russian), by I. B. Egiazarov, *Izvestiya Nauchnolssledovatelskovo Instituto Hydrotechniki*, No. 19, 1936, pp. 105–138.

(23) **"L'Utilisation de L'Air pour les essais de machines hydrauliques"** (Use of Air in Testing Hydraulic Machines), by C. Keller, *Le Genie Civil*, April 30, 1938, pp. 371–374.

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(25) **"Distorted River Models,"** by A. Rohringer.

(26) **"Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebewegung,"** by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Berlin, 1936.

Notation.—The following letter symbols, adapted for use in the second series of the Hydraulic Translations, conform essentially to American Standard Letter Symbols for Hydraulics,[1] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

A = cross-sectional area (Lauffer); the cross-sectional area of the entire mixture of water and air (Ehrenberger);
b = channel width (Ehrenberger): db_s = elementary width of channel at a point s;
C = constant or coefficient (Eq. 7, Lauffer):
  C_A = the proportion of the entire cross section occupied by water (Ehrenberger);
  C_w = the proportion of the cross-sectional area of a foamy mixture occupied by water (Ehrenberger);
c = coefficient (Lauffer):
  c_i = impulse coefficient;
  c_v = coefficient of velocity;
D = diameter; standard grain size (Rohringer), the subscript p denoting "prototype" and the subscript m denoting "model";
$\mathbf{F}$ = Froude's number;
g = acceleration of gravity;
H = average velocity head over the entire section (Ehrenberger);
h = head on a weir (Lauffer):
  h_m = velocity head at a point in the foamy layer (Ehrenberger);
  h_w = velocity head at a point in the stream (Ehrenberger);
  h_1 = reading on a manometer connected to hole 1; similarly for holes 2, 3, etc. (Krisam);
i = an exponent used for typographic simplicity (Lauffer);
k = a coefficient (of proportionality) in Eqs. 12 (Ehrenberger):
  k_1 = a coefficient applying to manometer 1 (Krisam); similarly for holes 2, 3, etc.;
  k_δ = a ratio depending on δ (Krisam, Eq. 9);
  $k(\min)$ = a subscript denoting the water surface for the smallest pressure and boundary layer between shooting and streaming flow (critical depth);
m = discharge coefficient for Bundschuh formula (Lauffer);

[1] ASA—Z10.2—1942.

n = Kutter's roughness coefficient;
o = a subscript to indicate a surface point (Lauffer);
P = pressure (Lauffer): P_w = measured air pressure in a water column;
p = unit pressure; gage pressure (Krisam, Lauffer):
p_b = pressure on a unit channel width (Lauffer);
p_o = pressure at the water surface (Lauffer);
p_∞ = gage reading for infinitely great velocity;
Q = quantity rate of flow, solid water (Ehrenberger): Q' = weighted flow ($= A\ V_m$) (Ehrenberger);
q = discharge per unit channel width: q(max) is a subscript denoting water surface for greatest discharge (Lauffer);
R = hydraulic radius for the entire cross section of flow, comprising air and water (Ehrenberger);
r = radius (Lauffer); also radius of pitot sphere (Krisam):
r_b = longitudinal radius of curvature of the stream bed;
r_c = radius of the graduated circle (Krisam);
r_l = radius of curvature of the stream lines;
r_m = mean radius of curvature (Lauffer);
S = slope, the subscript p denoting "prototype" and the subscript m denoting "model" (Rohringer): $S_o = \tan^{-1}\alpha$ = slope of the channel bottom;
s = a subscript denoting a point on the stream bed (Lauffer);
V = mean velocity of flow at a section $= \frac{Q}{A}$:
V_A = mean velocity on the basis of the total area of air and water (Ehrenberger);
ΔV = decrease in velocity due to air resistance (Ehrenberger);
v = velocity of flow at any point of a flowing stream (Ehrenberger):
v_a = axial component of velocity (Krisam);
v_m = mean velocity of flowing mixture of air and water (Ehrenberger);
v_o = velocity of flow at the surface as measured by floats;
v_r = radial component of velocity (Krisam);
v_u = tangential component of velocity (Krisam);
v_∞ = velocity at infinity (Krisam);
$\bar{v}$ = average velocity;
v' = velocity of flow for a given strip;
X = curvature constant (Lauffer);
x = distance measured parallel to the x-axis; also distances measured parallel to the y-axis (Krisam);
y = depth perpendicular to the stream bed (Ehrenberger):
y_e = elevation of the energy gradient above the stream bed;
y_m = average depth at a cross section (Ehrenberger); also normal distance of a point from the stream bed (Lauffer);
y_n = normal distance of a point from the water surface;
y_t = water depth for horizontal flow;

y'' = depth measured at one side of Rütz wasteway;
y_{aver} = average depth (Rohringer);
z = height of weir crest; distance measured parallel to the z-axis;
α = angle of chute: $S_o = \tan \alpha$;
γ = specific weight of water: γ_∞ = specific weight at infinite velocity;
δ = angle between the direction of the velocity and the axis through manometer hole 2 (Krisam);
θ = angle between two planes in a pitot sphere (Krisam, Fig. 1);
Φ = integration constant (Lauffer, Eq. 10); and
ϕ = velocity potential for the flow around a sphere of an ideal fluid (Krisam).

FLOW IN CHANNELS WITH VERTICALLY CURVED BEDS [1]

BY HARALD LAUFFER,[2] ESQ.

TRANSLATED AND ABSTRACTED BY CLARENCE E. BARDSLEY,[3] M. AM. SOC. C. E.

INTRODUCTION

The theoretical consideration of flow in open channels is concerned chiefly with the movement of water in horizontal or slightly sloped flumes, wherein potential flow occurs with flow lines parallel to the bed of the flume.

The outstanding exhaustive researches of Rehbock, Böss, Koch, Kozeny, and others have done much to bridge the hiatus between theoretical hydrodynamics and empirical hydraulics. It is quite possible that the energy gradient and the water-surface level or the maximum discharge can be ascertained under these assumptions since numerous comparable experiments have given extraordinary agreement. Therefore, calculations can be made with certainty. The writer has experimented with parallel flow utilizing greater slopes so that due consideration can be given the divergence of an unequal cross-sectional velocity distribution for corresponding potential flow.

It can be shown by many experiments, however, that under such circumstances the dynamic effects of flow-line curvature can become appreciable, and this influence cannot be neglected. Naturally, for flow in a flume cross section with a vertically curved bottom, it is of still greater significance.

In many cases, however, at least an approximation of the type of flow is worth striving for in advance, without waiting for the results of long experiments with models. Very often it is also important to become acquainted, in some degree, with the definite hydraulic magnitude of the bounding limits of the influences of curvature. In the following, an experiment is made in which the flow will be investigated through a flume with a vertical, curved bottom. In harmony with the customary procedure for streamlining, one must trace the progress of the streamline and verify the results experimentally.

The investigations embrace the following subjects: Pressure and velocity distribution; relation between the discharge, energy, and hydraulic gradients; discharge coefficients for round-crested weirs; and the flow régime.

DIFFERENTIAL EQUATIONS

To investigate the flow characteristics in channels with curved beds, one begins with the differential equation of pressure distribution in a representative

[1] "Strömung in Kanälen mit gekrümmter Sohle," by Harald Lauffer, *Wasserkraft und Wasserwirtschaft*, Vol. 31, October 1, 1936, pp. 249 and 260.

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cross section. Assume a stationary plane perpendicular to a stream tube and consider a stream element of unit width perpendicular to the direction of flow (Fig. 1). The water velocity at the point under consideration is V, and the radius of curvature of the streamline is r_l (positive when the center of curvature

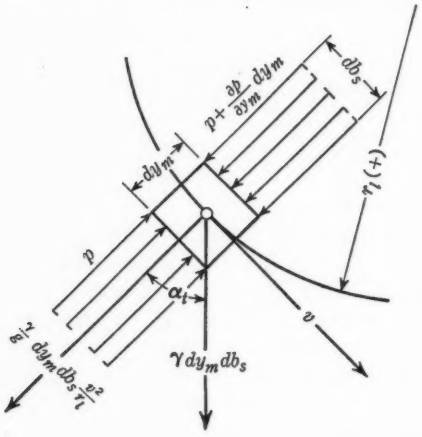

FIG. 1.—FORCES ON A STREAM ELEMENT NORMAL TO THE DIRECTION OF FLOW

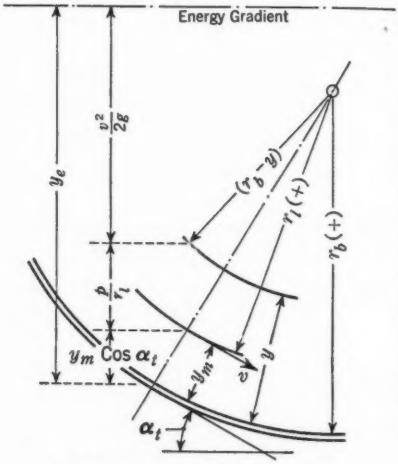

FIG. 2.—STREAMLINE CURVATURE AND ENERGY VARIATION

lies above the point). The external forces acting independent of friction are the force of gravity (which must be kept in equilibrium with the pressure forces) and the inertia forces.

In the direction of the normals to the streamlines—

$$\frac{\partial p}{\partial y_m}\,dy_m\,db_s + \gamma\,dy_m\,db_s\cos\alpha_t + \frac{\gamma}{g}\,dy_m\,db_s\,\frac{V^2}{r_l} = 0 \quad \ldots\ldots (1)$$

or

$$\frac{\partial\left(\frac{p}{\gamma}\right)}{\partial y_m} + \cos\alpha_t + \frac{V^2}{g\,r_l} = 0 \quad \ldots\ldots (2)$$

A suitable analytical function must be introduced for the radius of curvature r_l. The flow in channels with curved outlines is an analogous problem, according to Böss, if the assumption is made that the flow lines follow concentric curves. The streamline curvature is then computed by the radius r_b from the bed (Fig. 2):

$$r_l = r_b - y_m \quad \ldots\ldots (3)$$

Assuming a potential flow, all of the water particles must possess the same energy content. Then, for every cross section, the energy gradient has a fixed distance y_e above the bed. The sum of the potential head, pressure head, and velocity head must be constant for every point in a cross section (Fig. 2); thus,

$$y_e = y_m\cos\alpha_t + \frac{p}{\gamma} + \frac{V^2}{2\,g} \quad \ldots\ldots (4)$$

Eqs. 3 and 4 were both obtained from Eq. 2—

$$\frac{\partial\left(\frac{p}{\gamma}\right)}{\partial y_m} + \cos\alpha_t + 2\,\frac{y_e - y_m\cos\alpha_t - \frac{p}{\gamma}}{r_b - y_m} = 0 \ldots\ldots\ldots\ldots (5)$$

—or finally:

$$\frac{\partial\left(\frac{p}{\gamma}\right)}{\partial y_m} - \frac{2\,p}{(r_b - y_m)\,\gamma} + \frac{2\,(y_e - y_m\cos\alpha_t)}{r_b - y_m} + \cos\alpha_t = 0 \ldots\ldots\ldots (6)$$

Pressure and Velocity Distribution

In the last section, the derived differential equation for pressure distribution is a linear differential equation of the first order. Its standard integral is:

$$\frac{p}{\gamma} = C\,e^{-i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

[in which i is equal to $\int \frac{2\,dy_m}{r_b - y_m}$ —Ed.] and

$$e^{-i} = \int \left[\frac{2\,(y_e - y_m\cos\alpha_t)}{r_b - y_m} + \cos\alpha_t\right] e^{i}\,dy_m$$

becomes:

$$\frac{p}{\gamma} = \frac{C}{(r_b - y_m)^2} + y_e - y_m\cos\alpha_t \ldots\ldots\ldots\ldots\ldots (8)$$

The integration constant is determined by the limiting condition that the pressure on the free surface is infinitesimal.

$$y_m = d\left(\frac{p}{\gamma}\right) = \Phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which Φ is nearly infinitesimal. Eq. 9 gives

$$C = (r_b - y)^2\,(y\cos c_v - y_e) \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Combining Eqs. 8 and 9, the pressure distribution in a normal cross section is:[4]

$$\frac{p}{\gamma} = y_e - y_m\cos\alpha_t - (y_e - y\cos\alpha_t)\left(\frac{r_b - y}{r_b - y_m}\right)^2 \ldots\ldots\ldots (11^*)$$

For straight-line, parallel flow, the known pressure equation, for $r_b = \infty$, is:

$$\frac{p}{\gamma} = (y - y_m)\cos c_v \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Eq. 11 can be converted into:

$$\frac{p}{\gamma} = (y - y_m)\cos\alpha_t + (y_e - y\cos\alpha_t)\left[1 - \left(\frac{r_b - y}{r_b - y_m}\right)^2\right] \ldots\ldots (13)$$

[4] Equation numbers marked with an asterisk (*) indicate essential design formulas.

The first term of Eq. 13 agrees with the pressure equation for flow without curvature; the second term exhibits the influence of streamline curvature on the pressure distribution.

Fig. 3 shows two flow examples, computed by Eq. 11 or Eq. 13, which give the pressure-distribution lines. For both examples the radius of curvature of

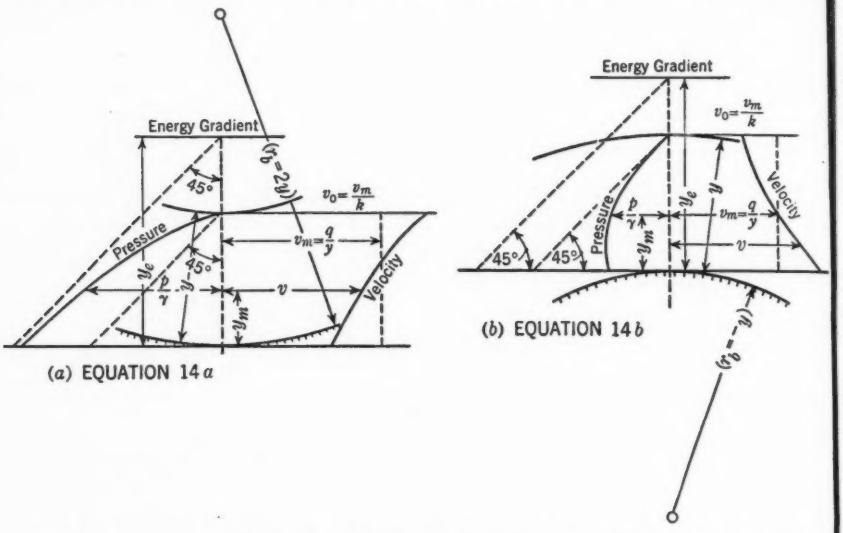

FIG. 3.—PRESSURE DISTRIBUTION AND VELOCITY DISTRIBUTION ACCORDING TO EQS. 12 AND 14

the average streamline is the same and equal to $r_m = 1.5\,y$, but with opposite algebraic sign. Also: $q = 2.5\,\frac{(y_m)^3\,(b_s)^{-1}}{y_m}$; $y = 1.0\,y_m$; $V_m = 2.5\,y_m\,(b_s)^{-1}$; and $\frac{(V_m)^2}{2\,g} = 0.318$. In Eq. 11: $r_b = +\,2.0\,y_m$; $\frac{y}{r_b} = +\,0.5$; $k = 0.693$; and

$$y_e = 1.0 + \frac{0.318}{0.693^2} = 1.66\,y_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

In Eq. 13: $r_b = 1.0\,y_m$; $\frac{y}{r_b} = -\,1.0$; $k = 1.387$; and

$$y_e = 1.0 + \frac{0.318}{1.387^2} = 1.166\,y_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

At times the given normal static pressure distribution shows the greater influence of the curvature.

The pressure-distribution lines are similar hyperbolic curves, whose asymptotes are, in the first case, the lines with the equation $\frac{p}{\gamma} = y_e - y_m \cos\alpha$, (comparable with the static pressure-distribution lines) and, in the second case, the lines parallel to the bed tangent through the center of curvature. A zero point, previously mentioned, lies in the water surface.

The velocity distribution is easily deduced from the pressure distribution and the energy theorem. From Eq. 4—

$$V = \sqrt{2\,g\left(y_e - y_m \cos\alpha_t - \frac{p}{\gamma}\right)} \quad \ldots\ldots\ldots\ldots\ldots (15a)$$

and with Eq. 11—

$$V = \sqrt{2\,g\,(y_e - y\cos\alpha_t)}\;\frac{r_b - y}{r_b - y_m} \quad \ldots\ldots\ldots\ldots\ldots (15b)$$

The surface velocity is then $y_m = y$—

$$v_o = \sqrt{2\,g\,(y_e - y\cos\alpha_t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15c)$$

By again substituting from Eq. 3:

$$V\,r_l = v_o\,r_{lo} = \sqrt{2\,g\,(y_e - y\cos\alpha_t)}\;(r_b - y)\;(= \text{a constant}) \ldots (16^*)$$

The velocity is also inversely proportional to the distance the given points are from the center of curvature. Therefore, this velocity distribution is characteristic for a potential flow along concentric curves.

The velocity curve is a hyperbola in a normal cross section. According to Eq. 16, the trace lines and the parallels to the bed tangents passing through the center of curvature form the asymptotes to the curves. At the center of curvature $y_m = 0$, and according to Eq. 15*b* the velocity becomes infinite. The center of curvature therefore must always remain outside of the actual fluid stream. In the region of the surface layers the condition is: $0 > \frac{r_b}{y} > 1$.

The proper velocity distribution is introduced in the flow examples shown in Fig. 3. It is also worthy of note that, under the assumed hypothesis, the velocity must always be positive. Therefore, zero velocity, or reversals of velocity, are excluded.

Discharge and Energy Gradient

From the velocity distribution according to Eq. 15*b*, compute the discharge per unit width of cross section:

$$q = \int_0^y v\,dy_m = \sqrt{2\,g\,(y_e - y\cos\alpha_t)}\;(r_b - y)\int_0^y \frac{dy_m}{r_b - y_m} \ldots (17a)$$

which finally gives,

$$q = y\,\sqrt{2\,g\,(y_e - y\cos\alpha_t)}\left(1 - \frac{r_b}{y}\right)\log\left(1 - \frac{y}{r_b}\right) \ldots\ldots (17b)$$

Introducing the curvature coefficient p_b—

$$p_b = \left(1 - \frac{r_b}{y}\right)\log\left(1 - \frac{y}{r_b}\right) \ldots\ldots\ldots\ldots\ldots (18^*)$$

and

$$q = v_o\,dp_b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19^*)$$

When the form of bed (r_b and α_t), the position of the energy gradient y_e, and the depth of the stream y are given, the discharge can be determined.

According to Eq. 19, the curvature coefficient p_b is the relation of the actual discharge to the discharge of a parallel flow with equal energy gradient height and stream thickness. In Fig. 4, the p_b-values are plotted on logarithmic paper and projected over several cycles. The limiting values of p_b for $\frac{y}{r_b}$ are always positive. For straight-line flow, $r_b = \infty$, $\frac{y}{r_b} = 0$, and $p_b = 1$. The two other limits are, for $\frac{y}{r_b} = +1$, $p_b = 0$; and, for $\frac{y}{r_b} = -\infty$, $p_b = \infty$.

The second problem is to determine the position of the energy gradient for a given flow (r_b, α_t, q, and y). The simplest energy gradient for a point on the surface can be computed on the assumption that the energy content of all water particles in a normal cross section is the same. From Eq. 4, it follows that, when $y_m = y$ and $p_o = 0$,

$$y_e = y_{eo} = y \cos \alpha_t + \frac{(v_o)^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and, from Eq. 19, $v_o = \frac{q}{dp_b}$. Therefore, the position of the energy gradient is

$$y_e = y \cos \alpha_t + \frac{1}{2g}\left(\frac{q}{dp_b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

In accordance with the usual method of expression, with $v_m = \frac{q}{y}$,

$$y_e = y \cos \alpha_t + \frac{v^2 y_m}{2g\,(p_b)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22^*)$$

The velocity head for average velocity is to be multiplied by $\frac{1}{(p_b)^2}$. The factor apparently entirely agrees with the coefficient of velocity, $c_v = \frac{\int_0^F v^3\, dA}{(v_m)^3\, A}$, according to Rehbock. It is also to be noted that $\frac{1}{(p_b)^2}$ takes into consideration not only the influence of the velocity distribution but also the pressure distribution, whereas a corresponding static pressure distribution was assumed for the coefficient c_v. Although the coefficient of velocity always must be greater than unity, the zero value of $\frac{1}{(p_b)^2}$ is $c_v \geqq 1$ and $0 < \frac{1}{(p_b)^2} < \infty$. In the flow examples (Fig. 3), the graph of the computed energy gradients, according to Eq. 42, shows again the greater influence of the streamline curvature. For corresponding p_b-values for equal values of y, q, and α_t, the energy gradient for convex flow always lies below, and for concave flow always above, the energy gradient for parallel flow.

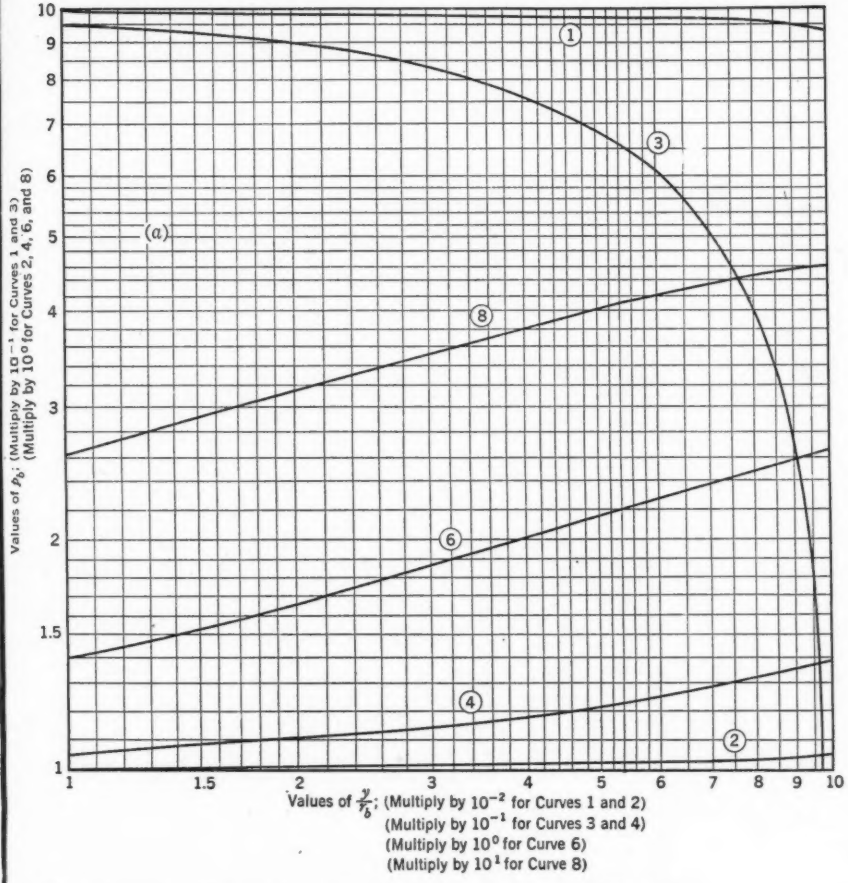
(a)
Values of p_b: (Multiply by 10^{-1} for Curves 1 and 3)
(Multiply by 10^{0} for Curves 2, 4, 6, and 8)
Values of $\frac{y}{r_b}$; (Multiply by 10^{-2} for Curves 1 and 2)
(Multiply by 10^{-1} for Curves 3 and 4)
(Multiply by 10^{0} for Curve 6)
(Multiply by 10^{1} for Curve 8)
(b)
Values of p_b
$\frac{y}{r_b}$ Negative
$\frac{y}{r_b}$ Positive
Values of $\frac{y}{r_b}$
$p_b = \left(1 - \frac{r_b}{y}\right) \mathrm{Log}\left(1 - \frac{y}{r_b}\right)$

FIG. 4.—CURVATURE COEFFICIENT $p_b = \frac{q}{v_0 y}$

Bed Pressure

The water pressure on the bed merits particular attention. Eq. 11 yields, for $y_m = 0$—

$$\frac{p_s}{\gamma} = y_e - (y_e - y\cos\alpha_t)\left(1 - \frac{y}{r_b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (23)$$

Expressed in dimensionless terms, with the new variables $\frac{y}{y_e}$ and $\frac{p_s}{\gamma\, y_e}$:

$$\frac{p_s}{\gamma\, y_e} = 1 - \left(1 - \frac{y}{y_e}\cos\alpha_t\right)\left(1 - \frac{y}{r_b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (24)$$

Eq. 24 makes possible the determination of the pressure on the bed for given bed shapes (r_b and α_t), the position of energy gradient, and stream depth, either analytically or graphically with the help of Fig. 5. In this figure a nest of curves gives various values of $\frac{y}{r_b}$, and another superimposed nest of curves gives values of $\frac{y_e}{r_b\cos\alpha_t}$.

If the term $(y_e - \cos\alpha_t)$ from Eq. 22 is substituted in the pressure equation (Eq. 13):

$$\frac{p}{\gamma} = (y - y_m)\cos\alpha_t + \frac{(v_m)^2}{2\,g\,(p_b)^2}\left[1 - \left(\frac{r_b - y}{r_b - y_m}\right)^2\right] \quad \ldots\ldots\ldots (25a)$$

whereupon the bed pressure for $y_m = 0$:

$$\frac{p}{\gamma} = y\cos\alpha_t + \frac{1}{2\,g}\left(\frac{q}{dp_b}\right)^2\left[1 - \left(1 - \frac{y}{r_b}\right)^2\right] \quad \ldots\ldots\ldots (25b)$$

Dividing through by $y\cos\alpha_t$, a dimensionless equation for the bed pressure is obtained:

$$\frac{p_s}{\gamma\, y\cos\alpha_t} = 1 + \frac{1}{2}\left(\frac{y_{t,\,gr}}{y\sqrt[3]{\cos\alpha_t}}\right)^3 \frac{2\,\frac{y}{r_b} - \left(\frac{y}{r_b}\right)^2}{(p_b)^2} \quad \ldots\ldots\ldots (26)$$

For critical depth and parallel flow, $y_{t,\,gr}$, for equal discharge, is:

$$y_{t,\,gr} = \sqrt[3]{\frac{q^2}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Eq. 26 permits the determination of the bed pressure for various curvatures for given discharges and stream thicknesses without first determining the position of the energy gradient. However, it is necessary to use Eq. 24 or Fig. 5.

Particular attention must be given each point in the figure where the bed pressure changes—that is, from positive excess pressure in regions of lower pressure. Caution must be used against the possibility of flooding the structure through siphoning action or the entrance of air, which would cause the flow to be discontinuous. This requires a new course of procedure. The zero points are easily determined from Fig. 5.

At the limiting value of low pressure, one must at times consider atmospheric pressure. A water pressure less than vapor pressure is not conceivable. The cavitation pressure is to be calculated by approximating this limit without allowing air to enter with the supply or discharge flow. In Fig. 5 this limiting

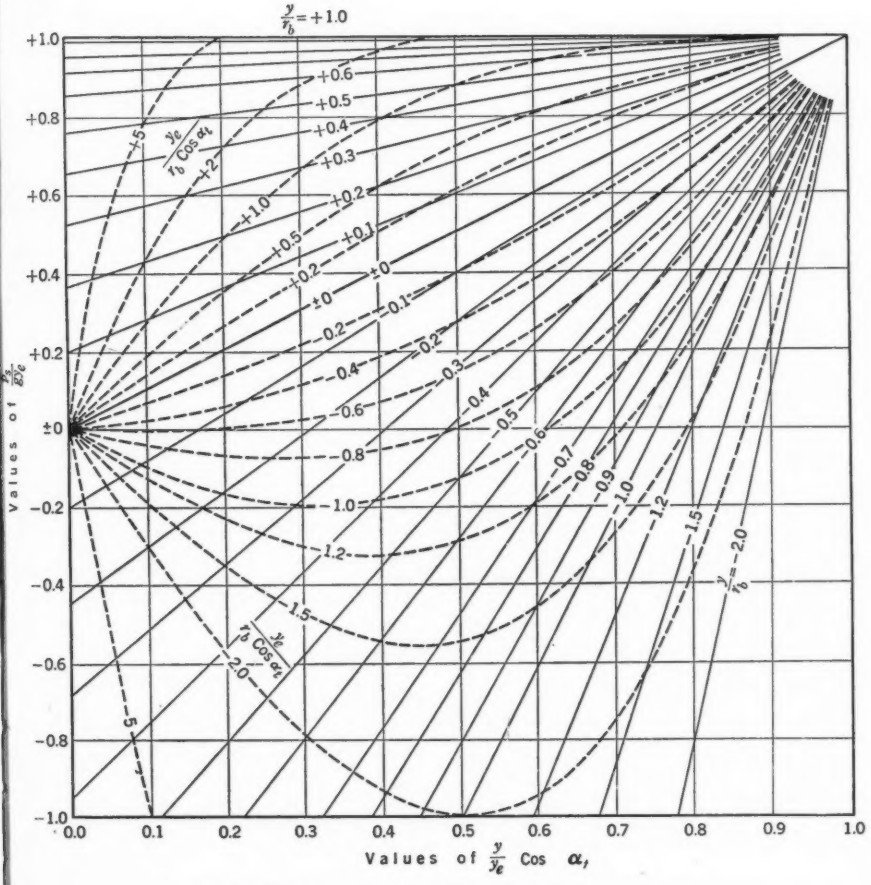

FIG. 5.—BED PRESSURE FROM POSITION OF ENERGY GRADIENT, STREAM THICKNESS, AND BED CURVATURE

case corresponds to a definite position of the energy gradient in a horizontal uniform flow, for which

$$\frac{p_s\,(\text{cav})}{\gamma\, y_e} = -\frac{P_w}{y_e} \qquad (28)$$

in which P_w = the measured air pressure in the water column. An investigation will be required only with very high water velocities.

DETERMINATION OF POSITION OF WATER SURFACE AND MAXIMUM DISCHARGE

To calculate the position of the water surface for a given discharge and position of the energy gradient, one must begin with the energy equation (Eq. 21):

$$y_e - y\cos\alpha_t = \frac{q^2}{2\,g\,y^2\,(p_b)^2} \quad \ldots\ldots(29)$$

which, with Eq. 18, gives

$$(y_e\,y^2 - y^3\cos\alpha_t)\left[\left(1-\frac{r_b}{y}\right)\log\left(1-\frac{y}{r_b}\right)\right]^2 = \frac{q^2}{2\,g} \quad \ldots\ldots(30)$$

Solving for y, the resulting equation is for the given values of q, y_e, and r_b. Corresponding to the derived procedure for parallel flow, one must transform Eq. 30 by changing the relation between the dimensionless magnitudes. Dividing through by $\frac{(y_e)^3}{\cos^2\alpha_t}$:

$$q_{gr} = \sqrt{g}\,(\tfrac{2}{3}\,y_e)^{1.5} \quad \ldots\ldots(31)$$

Finally,

$$\left[\left(\frac{y\cos\alpha_t}{y_e}\right)^2 - \left(\frac{y\cos\alpha_t}{y_e}\right)^3\right]\left[\left(1-\frac{r_b}{y}\right)\log\left(1-\frac{y}{r_b}\right)\right]^2 = \frac{4}{27}\left(\frac{q\cos\alpha_t}{q_{gr}}\right)^2 \quad \ldots\ldots(32)$$

From the identity,

$$\frac{r_b}{y} = \frac{r_b\cos\alpha_t}{y_e}\,\frac{y_e}{y\cos\alpha_t} \quad \ldots\ldots(33)$$

the q-linear equation is obtained as follows:

$$\left[\left(\frac{y\cos\alpha_t}{y_e}\right)^2 - \left(\frac{y\cos\alpha_t}{y_e}\right)^3\right] \times\left[\left(1-\frac{r_b\cos\alpha_t}{y_e}\,\frac{y_e}{y\cos\alpha_t}\right)\log\left(1-\frac{y_e}{r_b\cos\alpha_t}\,\frac{y\cos\alpha_t}{y_e}\right)\right]^2 = \frac{4}{27}\left(\frac{q\cos\alpha_t}{q_{gr}}\right)^2 \quad \ldots\ldots(34^*)$$

which is a function of the new variables $\frac{y_e}{r_b\cos\alpha_t}$, $\frac{y\cos\alpha_t}{y_e}$, and $\frac{q\cos\alpha_t}{q_{gr}}$.

Eq. 34 is seemingly a step backward in comparison with the simpler Eq. 30, and its analytical solution is of equal difficulty after $\frac{y\cos\alpha_t}{y_e}$ is eliminated. In its useful range, however, one can solve graphically for each of the three variables with the representative nest of curves (Fig. 6). The discharge will become zero for $\frac{y\cos\alpha_t}{y_e} = 0$ and $\frac{y\cos\alpha_t}{y_e} = 1$. The second expression, of course, is of value only for $\frac{y_e}{r_b\cos\alpha_t} \leqq 1$. The coefficient of curvature for

$$\frac{y}{r_b} = \frac{y\cos\alpha_t}{y_e}\,\frac{y_e}{r_b\cos\alpha_t} = 1 \quad \ldots\ldots(35)$$

disappears because the zero point for $\frac{y_e}{r_b \cos \alpha_t} > 1$, according to Eq. 35 for the position of the water surface, is:

$$\frac{y \cos \alpha_t}{y_e} = \frac{1}{\frac{y_e}{r_b \cos \alpha_t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

In these cases the center of curvature lies below the energy gradient and (see heading, "Pressure and Velocity Distribution") does not lie in the actual

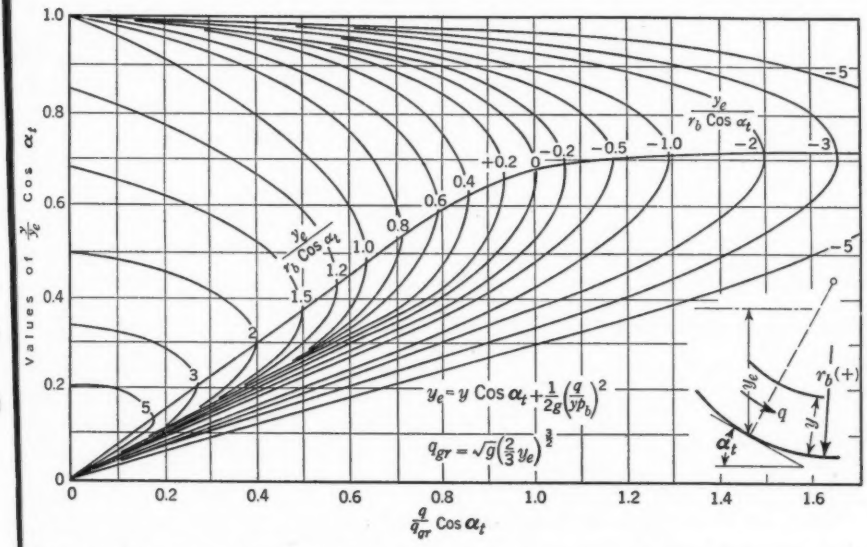

FIG. 6.—*q*-CURVES FROM EQ. 33 FOR DETERMINING THE POSITION OF THE WATER SURFACE FOR A GIVEN DISCHARGE AND ENERGY GRADIENT HEIGHT

fluid stream. Positions of the water surface above this limit are not conceivable according to Eq. 36.

The maximum points on the *q*-curves for equal curvature $\left(\frac{y_e}{r_b \cos \alpha_t} = \text{constant}\right)$ are determined with the aid of Eq. 34. The solution of the differential quotient seems hardly possible. Nevertheless, one may begin with Eq. 30 and write

$$\frac{\partial q^2}{\partial y} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The solution for y_e is then an easy one and gives the corresponding contracted expression:

$$y_{eq}(\max) = y \cos \alpha_t \frac{\left(1.5 - 1.5 \frac{r_b}{y}\right) \log \left(1 - \frac{y}{r_b}\right) + 1}{\log \left(1 - \frac{y}{r_b}\right) + 1} \quad \ldots\ldots (38)$$

With reintroduction of p_b from Eq. 18, the position of the water surface for the maximum discharge is:

$$\left(\frac{y\cos\alpha_t}{y_e}\right)_{q(\max)} = \frac{1}{1+\dfrac{0.5\,p_b}{\log\left(1-\dfrac{y}{r_b}\right)+1}} \quad \ldots\ldots\ldots (39)$$

For the determination of the corresponding discharge, introduce y_e from Eq. 38 into Eq. 30, and, after making a transformation, obtain—

$$q^2(\max) = g\,y^3\cos\alpha_t \frac{(p_b)^3}{\log\left(1-\dfrac{y}{r_b}\right)+1} \quad \ldots\ldots\ldots (40a)$$

Dividing by

$$\frac{q^2_{gr}}{\cos^2\alpha_t} = \frac{g\left(\frac{2}{3}\,y_e\right)^3}{\cos^2\alpha_t} \quad \ldots\ldots\ldots (40b)$$

gives

$$\frac{q(\max)\cos\alpha_t}{q_{gr}} = \left(1.5\,\frac{y}{y_e}\cos\alpha_t\right)^{1.5}(p_b)^{1.5}\left[\log\left(1-\frac{y}{r_b}\right)+1\right]^{-0.5} \quad \ldots (40c)$$

Substituting the value $\frac{y\cos\alpha_t}{y_e}$ from Eq. 39:

$$\frac{q(\max)\cos\alpha_t}{q_{gr}} = \left(\frac{1.5\,p_b}{1+\dfrac{0.5\,p_b}{\log\left(1-\dfrac{y}{r_b}\right)+1}}\right)^{1.5}\left[\log\left(1-\frac{y}{r_b}\right)+1\right]^{-0.5} \quad \ldots (41)$$

Hence, the investigator knows the position of the water surface and quantity for maximum discharge only as a function of $\frac{y}{r_b}$, whereas, to construct the maximum points of the q-curves, he must calculate these values as functions of $\frac{y_e}{r_b\cos\alpha_t}$. From Eqs. 39 and 41 the corresponding values of $\frac{y}{y_e}$, $\frac{y_{q(\max)}\cos\alpha_t}{y_e}$, and $\frac{q(\max)\cos\alpha_t}{q_{gr}}$ are computed.

The relation,

$$\frac{y}{r_b}\,\frac{y\cos\alpha_t}{y_e} = \frac{y_e}{r_b\cos\alpha_t} \quad \ldots\ldots\ldots (42)$$

then yields the corresponding value of $\frac{y_e}{r_b\cos\alpha_t}$. On the basis of these calculations, one can draw in Fig. 7 the curves for maximum discharge and the position of the water surface as a function of $\frac{y_e}{r_b\cos\alpha_t}$. Their values for a definite $\frac{y_e}{r_b\cos\alpha_t}$-value, corresponding to particular q-curves in Fig. 6, are then easy to find, the equation for y_e being $y_e = y\cos\alpha_t + \frac{1}{2g}\left(\frac{q}{dp_b}\right)^2$.

As already mentioned, the most important conclusion is the determination of the position of the water surface in a normal cross section with known bed (r_b and α_t) for a given discharge and position of the energy gradient y_e. From the position of the energy gradient, one can compute:

$$q_{gr} = \sqrt{g}\,(\tfrac{2}{3}\,y_e)^{1.5} \qquad (43)$$

and also the two quantities $q\,\frac{q}{q_{gr}}\cos\alpha_t$ and $\frac{y_e}{r_b\cos\alpha_t}$.

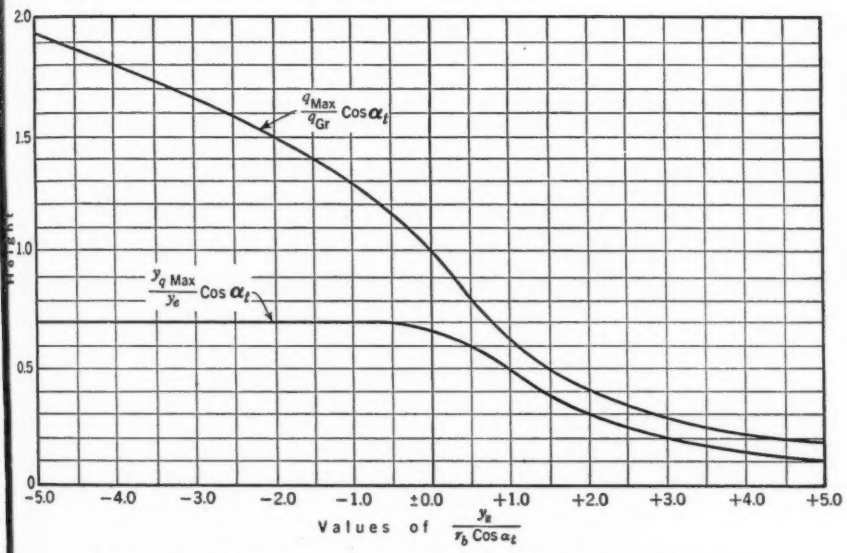

FIG. 7.—WATER-SURFACE ELEVATION AND QUANTITY FOR MAXIMUM DISCHARGE

The corresponding value of $\frac{y\cos\alpha_t}{y_e}$, and therefore the stream depth y, is obtained from Fig. 6. From the form of the q-curves, in general, two positions of the water surface are possible: $y_1 < y_{q(max)} < y_2$; that for $q = q(\max)$ is coincident. For $q > q(\max)$ the energy gradient height y_e is too small; hence, the energy gradient must then be elevated to discharge the given water quantity.

MEASUREMENT OF VELOCITY AND PRESSURE IN THREE-DIMENSIONAL FLOW [1]

By F. F. Krisam,[2] Esq.

Translated and Abstracted by Edward F. Wilsey,[3] Assoc. M. Am. Soc. C. E.

Synopsis

Ordinary types of pitot tubes are unsatisfactory for measuring velocity and pressure in three-dimensional flow. The 5-hole Zijnen sphere developed by Van der Hegge Zijnen[4] is well adapted to such measurements.

Description and Calibration of the Pitot Sphere

The instrument consists of the sphere, a (see Fig. 1), the shaft, b, and the graduated circular plate, c. The five holes are located on two great circles

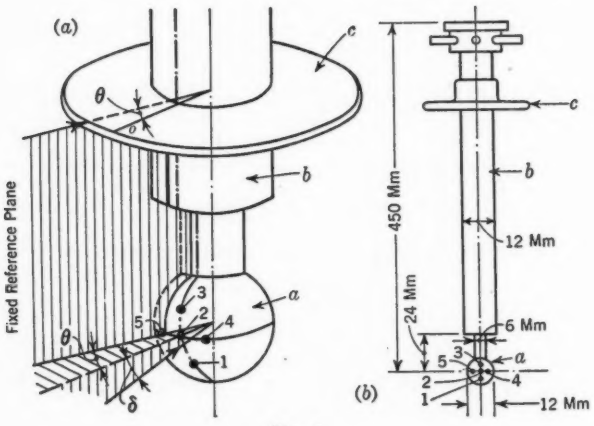

Fig. 1

normal to each other. The zero on the circular plate and holes 1, 2, and 3 lie in the same plane. Holes 4 and 5 are equidistant from hole 2.

In use, the sphere is rotated until the manometers connected to holes 4 and 5 indicate the same reading. In this position the plane passing through holes

[1] "Über die Messung von Geschwindigkeit und Druck in einer dreidimensionalen Strömung," by F. F. Krisam, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 23, No. 13, July 14, 1932.

[2] Dr.-Ing., Technical Univ., Karlsruhe, Germany.

[3] Asst. Engr., U. S. Bureau of Reclamation, Athens, Ohio.

[4] "Improved Instrument for the Simultaneous Determination of the Static Pressure, the Magnitude and Direction of the Velocity of Fluids," by Van der Hegge Zijnen, *Proceedings*, Royal Akademie Amsterdam, Vol. XXXII, No. 5, 1929; and "Verbetered instrument ter bespaling van den statischen druk, de grotte en richting der stromsnelheid van vloeistoffen," by Van der Hegge Zijnen, *Der Ingenieur*, Vol. 44, No. 33, 1929.

1, 2, and 3 lies in the direction of the flow. The angle θ between this meridian plane and a fixed datum plane is read on the graduated circle.

Let h_1, h_2, h_3, etc., be the readings on the manometers connected to holes 1, 2, 3, etc., respectively. Then—

$$\left.\begin{aligned} h_1 &= \frac{p}{\gamma} + k_1 \frac{v^2}{2g} \\ h_2 &= \frac{p}{\gamma} + k_2 \frac{v^2}{2g} \\ &\cdots\cdots\cdots \\ h_5 &= \frac{p}{\gamma} + k_5 \frac{v^2}{2g} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: $\frac{p}{\gamma}$ = the pressure head; v = the velocity; and k_1, k_2, etc. = the coefficients of the holes.

It is assumed that the coefficients k are solely dependent on the angle, δ, the angle between the direction of the velocity and the axis through hole 2. Because of this dependence, it must be possible to compute δ from the manometer readings.

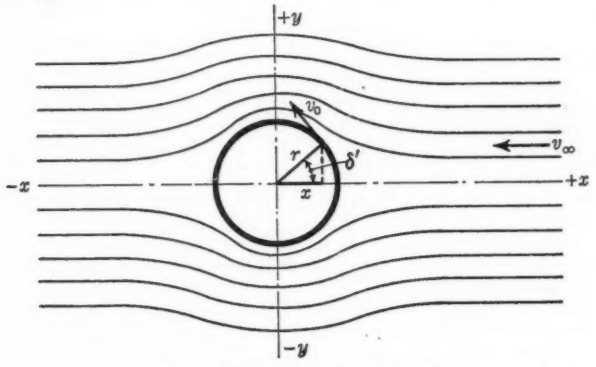

FIG. 2.—FLOW AROUND A SPHERE

If the coefficients of the holes have been determined previously, the velocity and pressure can be computed from:

$$h_1 - h_2 = \frac{v^2}{2g}(k_1 - k_2) \ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

$$\frac{v^2}{2g} = \frac{h_1 - h_2}{k_1 - k_2} = \frac{h_2 - h_3}{k_2 - k_3} = \cdots \ldots\ldots\ldots\ldots\ldots (2b)$$

and

$$\frac{p}{\gamma} = h_1 - k_1 \frac{v^2}{2g} = h_2 - k_2 \frac{v^2}{2g} = \cdots \ldots\ldots\ldots\ldots (2c)$$

The coefficients may be determined experimentally or from the theory of hydrodynamics applied to the flow of an ideal fluid around a sphere. The

velocity-potential for the flow around a sphere of an ideal fluid whose velocity at infinity is v_∞ may be written:

$$\phi = v_\infty x \left[1 + \frac{(r_c)^3}{2r^3}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

[The notation is as shown in Fig. 2, r_c being the radius of the graduated circular plate (Fig. 1). A more complete treatment of flow around a sphere is available elsewhere[5]—Ed.]

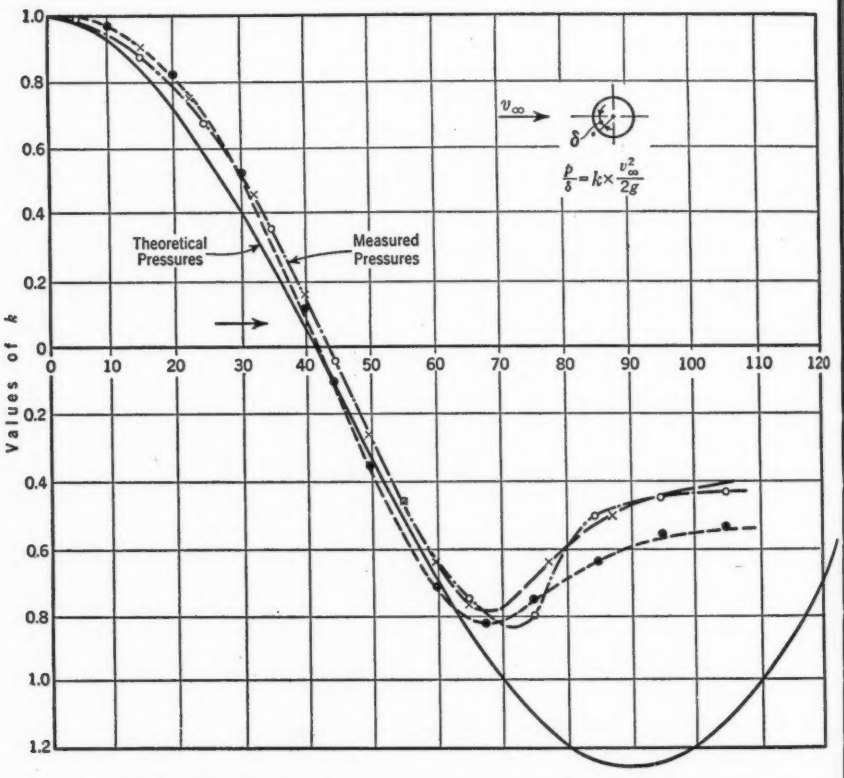

FIG. 3.—THEORETICAL AND ACTUAL PRESSURE DISTRIBUTION ON THE SPHERE

The components of the velocity may be found by differentiating Eq. 3. The resultant velocity is given by:

$$v^2 = (v_x)^2 + (v_y)^2 + (v_z)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The expression for the velocity on the surface of the sphere where $r = r_c$ may be shown to be:

$$v^2 = (v_\infty)^2 \frac{9}{4}\left(1 - \frac{x^2}{r^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

[5] "Aerodynamic Theory," edited by W. F. Durand, Julius Springer, Berlin, 1934, pp. 213–214.

By applying Bernoulli's theorem, the pressure head on the surface of the sphere may be written:

$$\frac{p}{\gamma} = \frac{p_\infty}{\gamma_\infty} + \frac{(v_\infty)^2}{2g}\left(\frac{9}{4}\frac{x^2}{r^2} - \frac{5}{4}\right) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The parenthetical expression in Eq. 6 is identical with the coefficient, k. Since $\cos \delta' = \frac{x}{r}$ (see Fig. 2),

$$k = \frac{1}{4}(9 \cos^2 \delta' - 5) \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

In Eq. 7, when $\delta' = 0$, $k = 1$; and, when $\cos \delta' = \sqrt{\frac{5}{9}}$, $k = 0$.

According to the theory, the pressure distribution on the back half of the sphere is identical with that on the front half. As a result of the separation of the boundary layer from the back half of the sphere, the pressure on the back half is only partly developed. However, on the front half the actual pressure distribution agrees well with the theoretical distribution because the influence of the friction is confined to a very thin layer. Fig. 3 shows the theoretical k-curve in comparison with several k-curves experimentally determined. In spite of the good agreement shown, the Zijnen sphere should be calibrated by experiment because of the influence of the shaft.

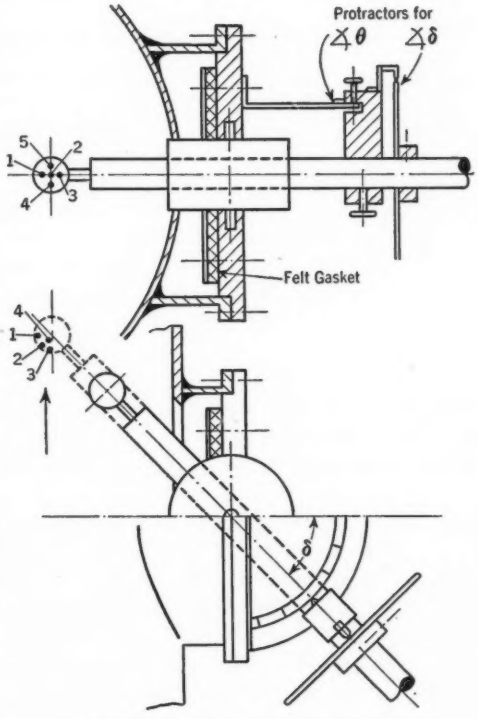

FIG. 4.—MOUNTING FOR CALIBRATION OF THE PITOT SPHERE

A pitot tube mounted for calibration in a long, straight pipe is shown in Fig. 4. A Prandtl pitot tube is placed in a symmetrical position, the symmetry of the velocity distribution having been previously demonstrated. The mounting was designed so that the entire instrument could be turned for measuring the angle, δ, and so that the sphere could be rotated for adjusting manometers 4 and 5 to the same reading. The shape of the curve for k_4 shows the influence of the location of hole 4, which lies at about 51° from hole 2. The interference of the shaft is reflected principally at holes 2 and 3,

which lie nearest to it. The shaft causes the pressure distribution to be shifted in the direction of $+\delta$. Fig. 5 shows the arrangement of the manometers for the calibration. The calibration should be made for a range of δ between $+60°$ and $-60°$, advancing 5° at a time.

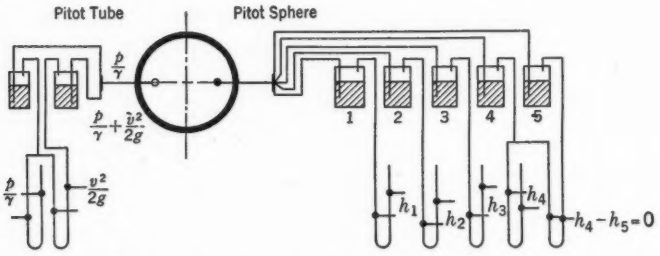

FIG. 5.—MANOMETER ARRANGEMENT FOR CALIBRATION

The coefficient for a single hole is computed from the values of $\frac{p}{\gamma}$ and $\frac{v^2}{2g}$ measured by an ordinary pitot tube. Thus,

$$\left.\begin{aligned} k_1 &= \frac{2g}{v^2}\left(h_1 - \frac{p}{\gamma}\right) \\ k_2 &= \frac{2g}{v^2}\left(h_2 - \frac{p}{\gamma}\right) \\ &\cdots\cdots\cdots \\ &\cdots\cdots\cdots \\ &\cdots\cdots\cdots \\ k_5 &= \frac{2g}{v^2}\left(h_5 - \frac{p}{\gamma}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Sample curves for these coefficients are shown in Fig. 6.

Let k_δ be a ratio which depends only on δ and which can be written in terms of the manometer readings. Such a ratio is:

$$k_\delta = \frac{k_3 - k_1}{k_2 - k_4} = \frac{h_3 - h_1}{h_2 - h_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

It is dependent on δ within the range from $-60°$ to $+60°$. If holes 1 and 4 were eliminated, the value of a sphere for determining δ would be very much reduced.

The unique dependence of the coefficients on δ implies that they are independent of the magnitude of the velocity. That this is the case is demonstrated by experiments on the resistance offered to fluid flow by spheres and cylinders (see Fig. 7). It is assumed that the pressure distribution remains unaltered, provided that the resistance of a sphere or cylinder remains constant over a range of velocities. Experiments[6] show that the resistance of a sphere is

[6] "Über den Luftwiderstand von Kugeln," by Wieselsberger, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, 1914; *Hütte*, 25th Ed., Vol. 1, p. 374; "Onderzoek betreffende de meetmethode met de Pitotbuis van dr. ir. J. J. Boren," by Meyer, *Der Ingenieur*, Vol. 43, No. 27, 1928; and "Abhandlungen aus dem Aerdynamischen," Institut der Technischen Hochschule Aachen, No. 5.

practically constant for Reynolds' numbers between 2×10^4 and 1.5×10^5. This range corresponds to air velocities from 26 to 200 m per sec and water

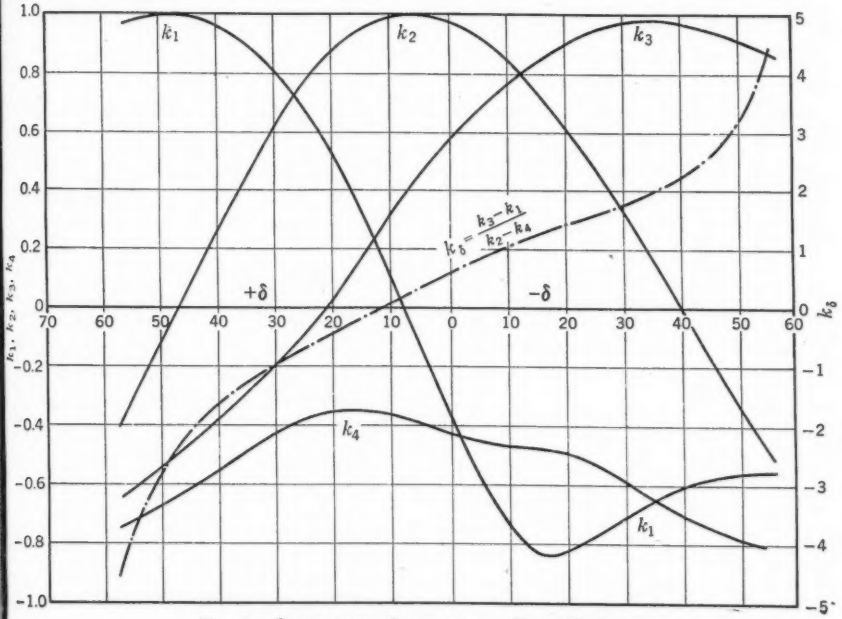

FIG. 6.—CALIBRATION CURVES FOR A PITOT SPHERE

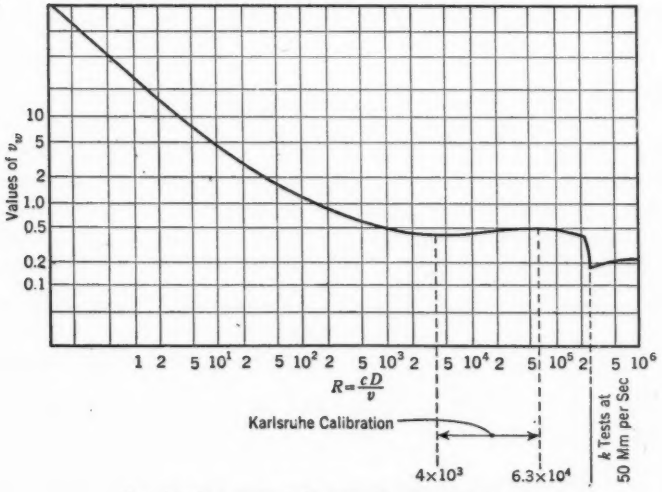

FIG. 7.—RESISTANCE COEFFICIENT OF THE SPHERE

velocities from 2 to 16 m per sec. However, between Reynolds' numbers from 2×10^3 and 2×10^4, there is only a small variation in the resistance.

Consequently, the lower limit may be put at 13 m per sec for air and 1 m per sec for water.

In order to obtain the greatest sensitivity, holes 4 and 5 should be located at points on the sphere where the pressure gradient is the maximum. Van der Hegge Zijnen has found that the slope of the pressure curve is a maximum at about 50° from hole 2.

Only a symmetrical pressure distribution on the front of the pitot sphere will justify its use as a measuring instrument. Krell's[7] experiments to the contrary notwithstanding, the measurements by Eisner[8] and Ermisch and at the Karlsruhe (Germany) laboratory have shown that this distribution is symmetrical at least on the part of the sphere where the flow is stable.

Sources of Error in Measurements with the Pitot Sphere

It may be objected that a measurement by a pitot sphere is not made at a point in the flow but includes an area of considerable magnitude. If the conduit in which the measurement is made is small, this objection is especially valid. It is extremely difficult to manufacture a pitot sphere smaller than 12 mm in diameter. However, a pitot tube shares this disadvantage to some extent.

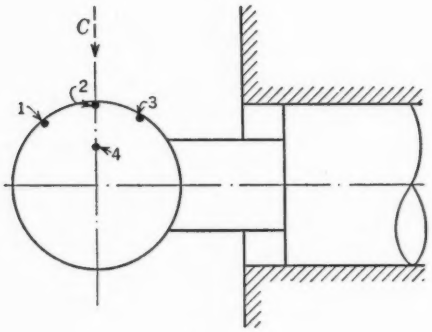

Fig. 8

The flow conditions in an experiment are usually more adverse than in a calibration. When the pitot sphere is applied to a flow having a steep pressure or velocity gradient $\left(\frac{\partial p}{\partial y} \text{ and } \frac{\partial v}{\partial y}, \text{ respectively}\right)$, errors will arise which can be evaluated only in certain special cases.

Finally, errors arising from the use of a pitot sphere close to a wall will be considered. In general, the coefficients of those holes nearest the wall are reduced. For example, in Fig. 8, k_3 is affected most and k_2 and k_4 to a lesser degree. The difference, $k_2 - k_4$, will not be affected much because both coefficients are reduced about in the same proportion. Since $\frac{v^2}{2g}$ is computed from $\frac{h_2 - h_4}{k_2 - k_4}$, the error arising from the proximity of a wall, as a rule, will not be large. Hole 1 is the most expedient to use in this case for computing the pressure, thus:

$$\frac{p}{\gamma} = h_1 - k_1 \frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

[7] "Druckverteilung an der luftumströmten Kugel," by Krell, Z.F.M., No. 4, 1931.

[8] "Widerstandsmessungen an umströmten Zylindern," by Eisner, *Mitteilungen Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Berlin, No. 4, 1929.

Sample Measurements

In Fig. 9 are shown the results of pressure and velocity measurements in the draft tube of a Kaplan turbine. In the first test the runner was removed and a set of radial guide vanes were installed in place of the gate apparatus. The

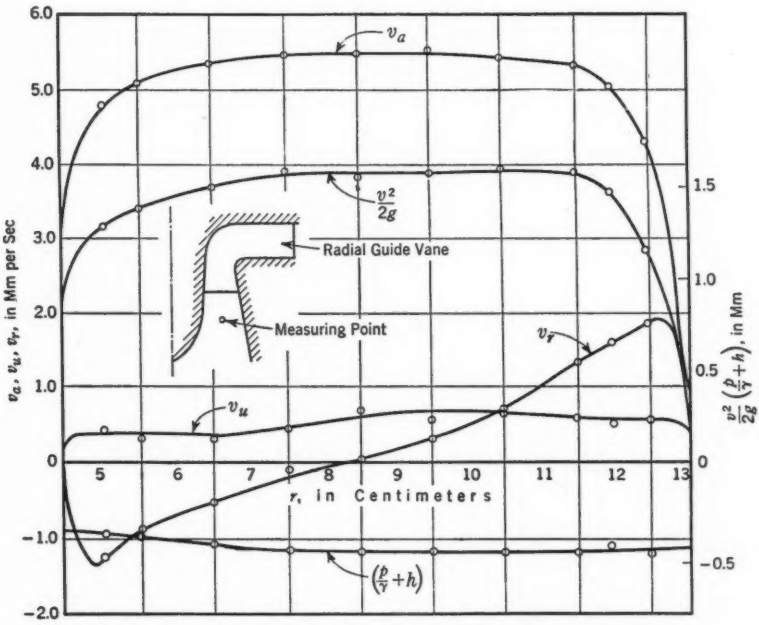

Fig. 9.—Velocity and Pressure Distribution in Meridional Flow

components of the velocity are found from the orientation of the pitot sphere (see Fig. 1), thus:

The axial component—

$$v_a = v \cos \delta \sin \theta \quad \ldots\ldots (11a)$$

the tangential component—

$$v_u = v \cos \delta \cos \theta \quad \ldots\ldots (11b)$$

and the radial component—

$$v_r = v \sin \delta \quad \ldots\ldots (11c)$$

The discharge computed from—

$$Q = 2\pi \int r\, v_a\, dr \quad \ldots\ldots (12)$$

gave 233.5 liters per sec (3,860 gal per min), whereas a weir measured 228 liters per sec (3,610 gal per min). This gives an error of about 2%. The velocity curves show that, in spite of the guide vanes, a small tangential component of velocity exists. Because of the sloping walls of the draft tube, there is a

positive radial component of velocity at the wall and a negative radial component at the hub of the runner.

In the second test (see Fig. 10), the discharge of a Kaplan turbine for a partial gate opening was measured. The location of the traverse was the

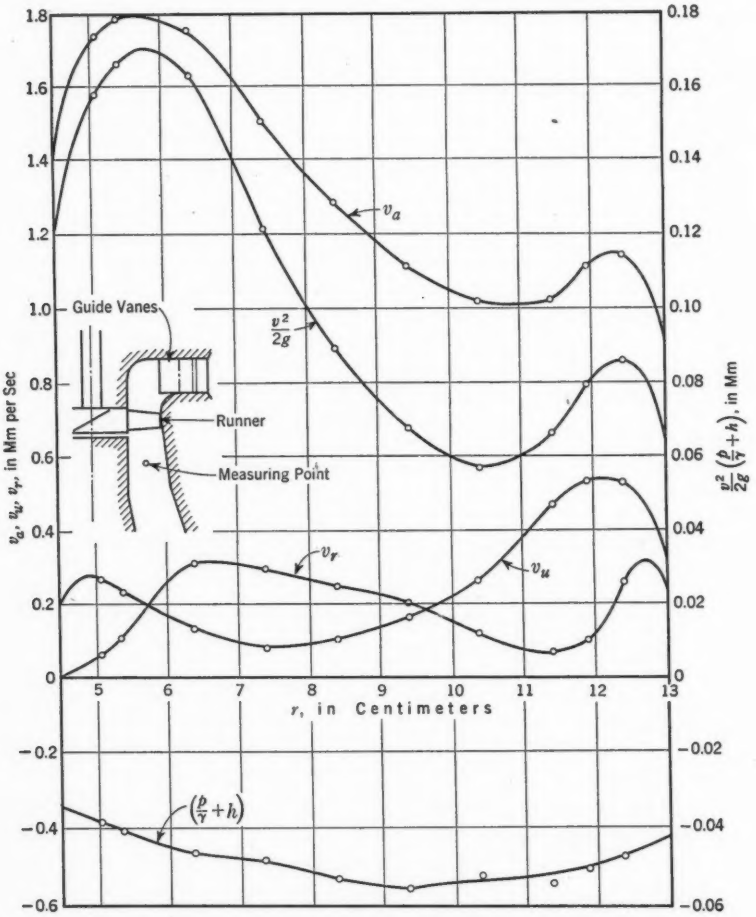

FIG. 10.—VELOCITY AND PRESSURE DISTRIBUTION BELOW A TURBINE RUNNER

same as in the first test. The discharge computed from the axial component of velocity differed from the weir discharge by only 0.8%. Other tests of this series gave comparable accuracies.

FLOW IN STEEP CHUTES WITH SPECIAL REFERENCE TO SELF-AERATION [1]

BY R. EHRENBERGER,[2] ESQ.

TRANSLATED AND ABSTRACTED BY EDWARD F. WILSEY,[3] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Rapid flow in steep chutes is subject to mechanical aeration as the result of the frictional resistance at the wetted perimeter and at the surface exposed to the atmosphere.

The purpose of this investigation was to establish a relation for the mean cross-sectional velocity in terms of the corresponding hydraulic radius and the inclination of the chute, and to devise a formula for the proportion of water in the aerated flow.

SCOPE OF THE INVESTIGATION

The experiments described herein were conducted in the Hydraulic Structures Research Institute in Vienna.

Five bottom slopes were tested—15.5, 20.6, 32.0, 49.5, and 76.2%. Because the available headroom in the laboratory was limited to 11.5 ft, the length of the chute was 52.5 ft for the smallest slope, decreasing to 18.0 ft for the steepest slope. Only one width of chute (0.82 ft) was included in the model tests. For each slope, the discharge was increased in three steps from 0.353 cu ft per sec to 1.57 cu ft per sec. Except in the case of the steepest chute, the length was sufficient for the establishment of uniform flow at the lower end of the chutes.

Experimental data on the Rütz wasteway in Austria were also included in this paper. This wasteway is constructed of wood on a slope of 76.2% and has a trapezoidal cross section whose bottom width is 8.2 ft.

MEASUREMENT OF DEPTH AND SURFACE VELOCITY

Water flowing down a steep chute appears as a very rough surface over which there is considerable spray. A water surface in the usual sense of the term does not exist, especially at the higher velocities. Fig. 1(*a*) shows a schematic cross section of the flow. At the top, small droplets of water are

[1] "Wasserbewegung in steilen Rinnen (Schusstennen) mit besonderer Berücksichtigung der Selbstbelüftung," by R. Ehrenberger, *Österreichischen Ingenieur- und Architektenvereines*, Nos. 15/16 and 17/18 1926, p. 155.

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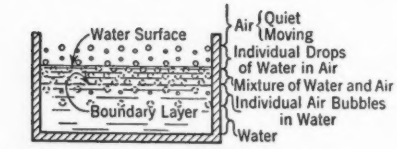
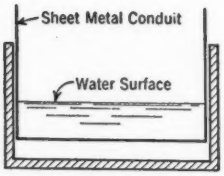
[3] Asst. Engr., U. S. Bureau of Reclamation, Athens, Ohio.

detached from the main body of the flow; below this there is a milky whit layer of water intimately intermingled with bubbles of air; then comes a laye of water containing relatively few air bubbles; and, finally, there is a layer clear water free from bubbles of air. This is the flow pattern as observe through glass windows placed along the sides of the chutes. The foamy laye extended all the way to the bottom of the chute for the two steepest slopes From this description, it is evident that the definition of depth is arbitrary



(*a*) SCHEMATIC CROSS SECTION OF FLOW



(*c*) CALIBRATION OF THE FLOW LAYERS (CROSS SECTION OF THE CHUTE)

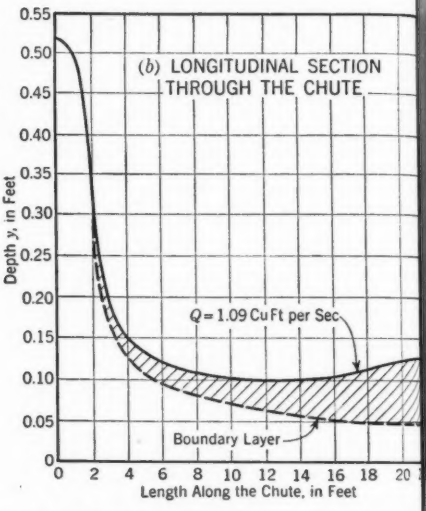


FIG. 1.—FLOW CHARACTERISTICS IN EXPERIMENTAL CHUTE

In what follows, the term depth ($= y$) refers to the perpendicular distanc from the bottom to the point at which the droplets of water in the uppermos layer rebound with a certain force from the broad side of a small bar hel perpendicular to the flow. The bar was so mounted that both longitudina and transverse water-surface profiles could be determined.

Fig. 1(*b*) shows the variation of depth along a longitudinal section of th chute. There is a more or less well-defined "boundary surface" between th foamy portion of the flow (crosshatched) and the relatively clear bottom flow The curve of depth shows a minimum point. Ordinary velocity-measurin devices, such as current meters and pitot tubes, are not applicable to th foamy portion of the flow because its density varies throughout the thickne of the layer. Pitot-tube measurements are valid, however, for that portio of the flow below the "boundary surface."

The "surface velocity" was obtained by photographing simultaneously float that consisted of a small wooden matchbox, to which a strip of burnin magnesium was attached, and a clock, the face of which was illuminated by 25,000-candle-power arc lamp, supplied with 50-cycle alternating curren The burning magnesium photographed as a solid line, but the hand of th clock was recorded as a series of isolated images. Twice in one complet

cycle of the alternating current, the intensity of the light emitted by the arc lamp varied from a maximum to zero. Therefore, in 1 sec there were 100 images of the hand on the clock, and each image was separated from its neighbors on either side by a time interval equal to 0.01 sec. The displacement of the float was determined from gage lines of known length placed every meter

TABLE 1.—RESULTS OF TESTS

Q (cu ft per sec)	Adjusted Values		R (ft) $b = 0.82$ ft	V (ft per sec)	$C_A = \frac{Q}{A V_A}$	H (ft)	$= \frac{C_A Q^2}{2 g H A^2}$
	$\bar{v}_o$	y_o (ft)					
(*a*) TEST SERIES NO. I (TAN $\alpha = 0.155$; SIN $\alpha = 0.153$)							
0.353[a]	10.66	0.057	0.050	9.61[b]	0.790	....	....
0.706	13.29	0.090	0.074	11.94	0.790	1.80	0.763
1.09	14.93	0.123	0.094	13.29	0.807	2.20	0.813
1.57	16.08	0.161	0.116	14.53	0.801	2.69	0.783
(*b*) TEST SERIES NO. II (TAN $\alpha = 0.206$; SIN $\alpha = 0.202$)							
0.353[a]	11.65	0.054	0.048	10.24[c]	0.770	....	....
0.706	14.93	0.087	0.072	13.06	0.750	2.13	0.701
1.09	16.73	0.120	0.093	14.70	0.750	2.59	0.728
1.57	18.05	0.156	0.113	16.04	0.758	3.02	0.762
(*c*) TEST SERIES NO. III (TAN $\alpha = 0.320$; SIN $\alpha = 0.305$)							
0.353	13.45	0.051	0.045	11.48	0.729	1.71	0.638
0.706	17.06	0.082	0.068	14.47	0.716	2.46	0.678
1.09	19.36	0.115	0.090	16.31	0.704	2.95	0.694
1.57	21.00	0.151	0.111	17.55	0.707	3.35	0.714
(*d*) TEST SERIES NO. IV (TAN $\alpha = 0.495$; SIN $\alpha = 0.444$)							
0.353[a]	15.42	0.048	0.043	13.22[d]	0.675	....	....
0.706	19.36	0.079	0.066	16.80	0.644	2.95	0.613
1.09	21.98	0.112	0.088	18.73	0.631	3.54	0.612
1.57[a]	24.05	0.148	0.109	20.18	0.635	....	....
(*e*) TEST SERIES NO. V (TAN $\alpha = 0.762$; SIN $\alpha = 0.606$)							
0.353	17.39	0.052	0.047	15.78	0.513	2.17	0.481
0.706	21.33	0.085	0.071	19.46	0.512	3.18	0.486
1.09	24.61	0.118	0.092	21.85	0.51	3.81	0.508
1.57[a]	26.87	0.156	0.113	23.62	0.513	....	....

[a] No pitot tube measurements made. [b] $V = 0.9\,\bar{v}_o$. [c] $V = 0.88\,\bar{v}_o$. [d] $V = 0.86\,\bar{v}_o$.

(3.28 ft) along the sides of the chute. The displacement of the float divided by the elapsed time gave the average velocity. Only in regions of uniform flow is this the velocity at a point. For this reason, several floats were released at the same time in the steepest chute to obtain the variation of velocity, for in this case uniform flow was not attained at the lower end of the chute. The "surface velocities" thus measured are average velocities over the width of the matchbox, or about one seventh the width of the chutes. The floats did not always move parallel to the center line of a chute. Hence, it was

necessary to reduce all "surface-velocity" measurements to velocities along the chutes.

These "surface velocities" more accurately represent the velocities at the bottom of the matchbox than at the surface. The proportion of water in the foamy layer C_A increases rapidly from the surface to the bottom of a chute. Therefore, the particles of water near the bottom of the floats play the major rôle in propelling the floats. The elevation of the top of a float was obtained by stretching a wire across a chute in such a position that the float just passed under the wire without touching. The experimental results, corrected by plotting the data in appropriate graphs and reading the values from the curves so obtained, are given in Table 1.

Distribution of Velocity and Proportion of Water in the Flow

The foamy mixture did not reach the bottom except in the case of the steepest chute. In the other tests, the velocity of the clear flow near the bottom was determined with a pitot tube. Curves of the velocity along a normal to the bottom were drawn using the pitot-tube data for the lower part of the curves and the velocity at the bottom of the floats for the one point on the upper part of the curves. Thus, the upper part is somewhat arbitrary. However, if the cross section is divided into horizontal strips and the mean velocity is computed from

$$V = \frac{\Sigma(C_w)' v'}{\Sigma(C_w)'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

the discharge computed from $A\ V_A$ agrees well with weir measurements. In Eq. 1, $(C_w)'$ and v' are the water proportion and the velocity, respectively, for the strip.

If C_A is the proportion of the area of the entire cross section occupied by water, the law of continuity for the flow in steep chutes may be written: $Q = A\ V_A\ C_A$; or

$$C_A = \frac{Q}{A\ V_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

An attempt was made to measure the discharge of water through a horizontal strip using a metal trough fitted to the inside of the chute (Fig. 1(*c*)). This trough could be moved up or down so as to isolate any part of the flow. However, the thin metal trough vibrated excessively, and the volumetric determination of the water discharge was complicated by the air retained in the flow. Therefore, this method for determining the distribution of air in a cross section was abandoned.

When a pitot tube is used in the foamy flow, the riser tubes do not contain this mixture of air and water because the air escapes rapidly. Since the riser tubes contain water, the true velocity head of the foamy mixture is given by

$$h_w = h_m\, C_w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

[in which h_w is the measured quantity—Ed.]. Hence, $h_m = \frac{v^2}{2g} = \frac{h_w}{C_w}$; or,

$$C_w = \frac{2\,g\,h_w}{v^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

To reduce the fluctuations of the water columns in the riser tubes, glass tubes 50 mm in diameter were used, but, in spite of their large size, pulsations persisted. The readings were taken after observing the fluctuating columns for several minutes. A complete traverse required four or five hours.

In Figs. 2 and 3 are shown the velocity-head curves along normals to the bottom and along horizontal lines, respectively, for a bottom slope of 20.6% and a discharge of 0.706 cu ft per sec. Fig. 2(*c*) is the average of five velocity-

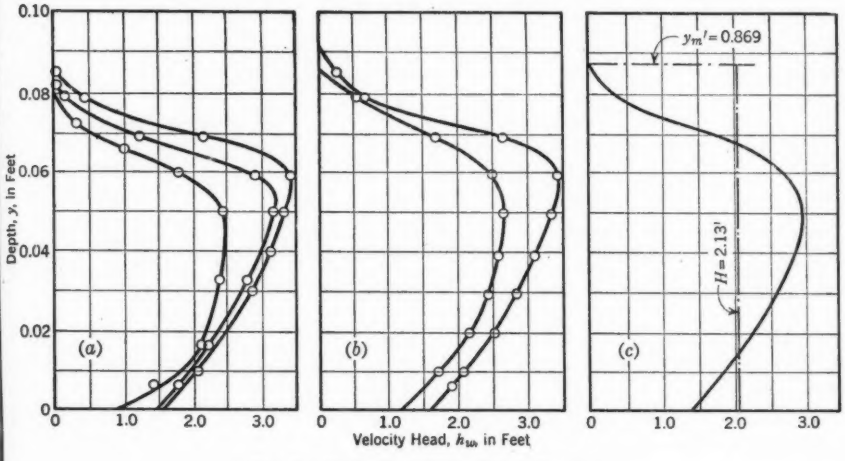


FIG. 2.—NORMAL VELOCITY-HEAD CURVES

head curves for the cross section. Fig. 4 shows an average velocity curve obtained as described previously. Using these two average curves and Eq. 4, values of the water proportion for horizontal strips can be computed. A curve showing the variation of the water proportion is also plotted in Fig. 4. It is seen that at the surface the proportion of water, C_w, is zero. As the depth increases, C_w increases rapidly until, at the "boundary surface," it is 100%. An average of C_w for the entire cross section can be computed from this curve.

If H and V_A represent average values of the velocity head and velocity, respectively, over the entire cross section, then C_A, the average proportion of water for the entire cross section, may be computed approximately from

$$C_A = \frac{2\,g\,H}{V^2} \quad \ldots\ldots (5)$$

From Eq. 2,

$$V = \frac{Q}{A\,C_A} \quad \ldots\ldots (6)$$

Hence,

$$C_A = \frac{Q^2}{2\,g\,H\,A^2} \quad \ldots\ldots (7)$$

Values of C_A computed from Eqs. 2 and 7 are given in Table 1. [Eq. 2 evaluates C_A on a basis of average velocities and Eq. 7 on a basis of average heads.—Ed.]

Measurements on the Rütz Wasteway

Three sets of measurements have been made on the Rütz wasteway, namely:

1. Three measurements by Rümelin and Angerer in 1913.
2. Four measurements by the Vienna Hydraulic Laboratory in 1923.
3. Five measurements by the W.A.G. (Water Power Company) in 1923.

Rümelin and Angerer measured the discharge with a weir, whereas in the other two sets of measurements the discharge was determined from the inflow

TABLE 2.—Measurement

No.	Description (1)	Rümelin and Angerer (2)		
	(a) Observed Measurements			
	Depths, in Feet:			
1	y''	....	....	....
2	$y_o = 0.8\ y''$	0.10	0.16	0.48
3	Hydraulic radius R, in feet[a]	0.097	0.157	0.44
4	Area A, in square feet[a]	0.82	1.40	4.2
	Velocities, in Feet per Second:			
5	v_o [b]	26.2	35.8	62.3
6	$v_m = 0.9\ v_o$	23.6	32.2	56.1
7	Flow Q, in cu ft per sec[c]	17.7	28.3	120.0
	(b) Adjusted Measurements and Computed Aeration			
8	Hydraulic radius R, in feet	0.097	0.157	0.44
9	Velocity V, in feet per second[d]	23.6	30.2	53.2
	Flow, in Cubic Feet per Second:			
10	Q [d]	12.4	18.7	86.5
11	$Q' = A\ v_m$	19.4	42.0	223.5
12	Water proportion $C_A = \frac{Q}{Q'}$	0.64	0.44	0.3

[a] Computed using y. [b] Measure

and outflow of the reservoir without taking into account any changes in storage. The outflow was determined from two current meter measurements in the second series and from the discharge curves of the turbines in the third series. The depth was measured at one side of the wasteway in the second and third series. Rümelin found that the transverse water-surface profile was concave and that the surface was about a third of a foot higher on one side than on the other when the discharge was 120 cu ft per sec. For this reason, the average depth, y_m, at a cross section was found from

$$y_m = 0.8\ y'' \qquad (8)$$

for the last two sets of measurements. In Eq. 8, y'' is the depth at one side.

The surface velocity, v_o, was determined by floats. The relation between the average velocity, V, over the cross section and the surface velocity, as

found from the tests on the model of the Rütz wasteway, was

$$V = 0.9\, v_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Eq. 9 was used in computing the Rütz measurements. Table 2 shows the data and computed results for the Rütz wasteway.

The data for the model tests and the adjusted data for the Rütz wasteway are plotted in Fig. 5. The broken lines through points indicated by circles apply to the model tests. The full lines apply to all of the data, and their equations are:

$$V = 97\, R^{0.52} \sin \alpha^{0.4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

ON THE RÜTZ WASTEWAY

Vienna Hydraulic Laboratory (3)				Water Power Company (4)					No.
....	....	....	....	0.33	0.47	0.49	0.56	0.92	1
....	....	....	....	0.26	0.38	0.39	0.45	0.74	2
0.210	0.272	0.361	0.512	0.249	0.348	0.364	0.407	0.640	3
1.83	2.42	3.29	4.90	2.22	3.18	3.35	3.80	6.37	4
39.4	45.3	52.5	63.3	44.0	53.5	53.2	55.8	69.2	5
35.5	40.8	47.2	57.0	39.6	48.2	47.9	51.2	62.3	6
27.9	36.0	52.3	129.2	32.8	56.6	62.5	70.3	151.9	7
0.210	0.272	0.361	0.512	0.249	0.348	0.364	0.407	0.640	8
35.4	40.7	47.2	57.1	38.7	46.6	47.6	50.5	64.0	9
26.5	37.1	57.6	113.0	31.8	54.7	58.3	72.4	176.6	10
65.0	98.5	155.7	279.3	85.8	148.7	159.3	192.5	406.8	11
0.41	0.38	0.37	0.40	0.37	0.37	0.37	0.38	0.43	12

[c] Water only. [d] Adjusted values.

and

$$C_A = 42\, R^{-0.05} \sin \alpha^{-0.26} \ (\sin \alpha < 0.476) \ldots\ldots\ldots\ldots (11a)$$

$$C_A = 30\, R^{-0.05} \sin \alpha^{-0.74} \ (\sin \alpha > 0.476) \ldots\ldots\ldots\ldots (11b)$$

It is seen that the lines through the data for the model tests deviate from the full lines. This may be attributed to the fact that the air resistance on the matchbox floats was neglected, with the result that the measured velocities at the surface are too small. The decrease in velocity, ΔV, due to air resistance on the matchbox, may be written:

$$\Delta V = k\, V^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

If the line for $\sin \alpha = 0.606$ for the model tests is prolonged upward and the values of the deviations between this line and the corresponding full line are

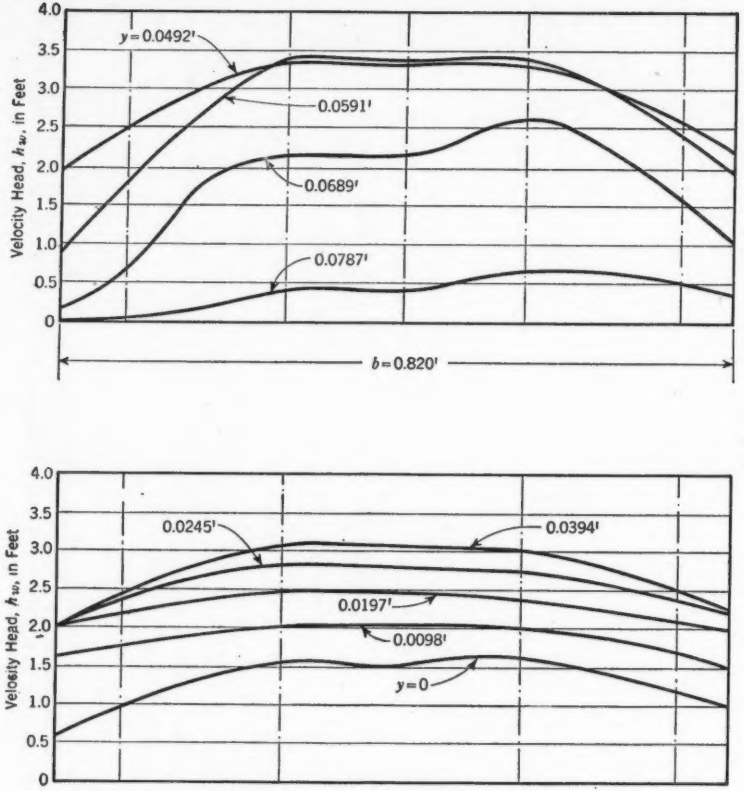


FIG. 3.—HORIZONTAL VELOCITY-HEAD CURVES

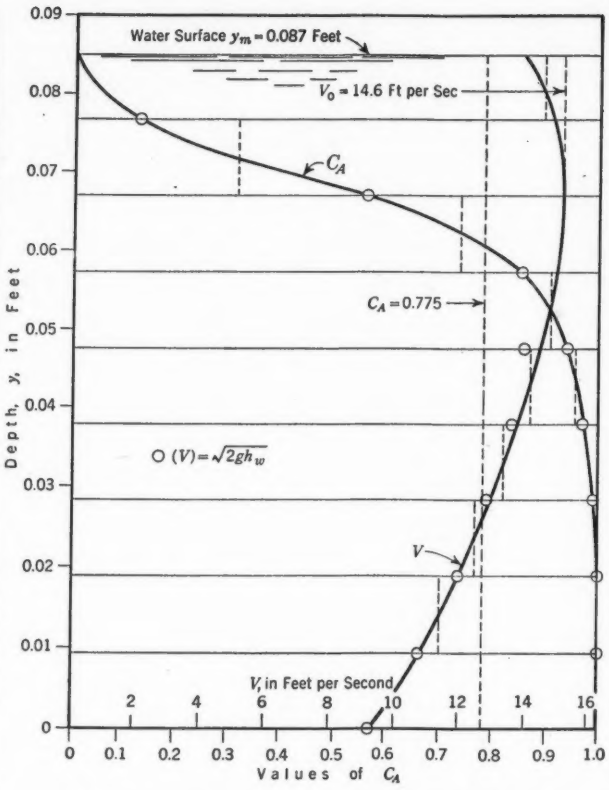


FIG. 4.—CONSTRUCTION OF THE y-CURVES AND C_A-CURVES

Hydraulic Radius, R, in Feet

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read for various values of the velocity, and, if the deviations are plotted against velocity on logarithmic paper, the following equation results:

$$\Delta V = k\, V^{1.6} \qquad (12b)$$

The fact that V has an exponent of 1.6 rather than 2 may be attributed to the movement of the air in the region of the float. Thus, the relative velocity

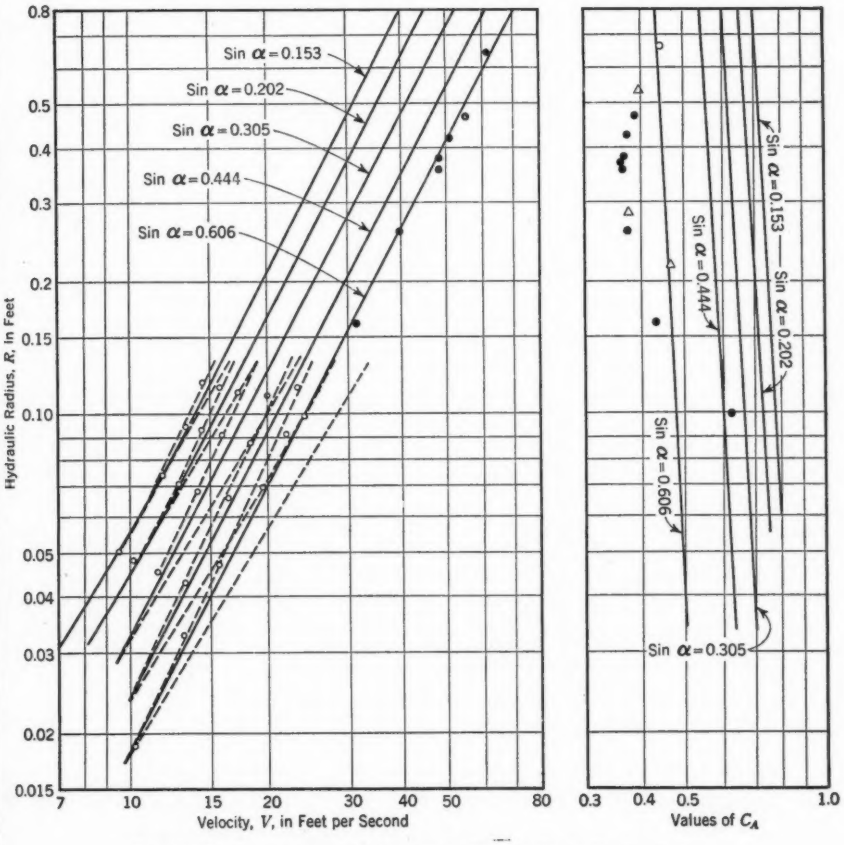

FIG. 5.—INFLUENCE OF THE HYDRAULIC RADIUS R ON V AND ON C_A

between the float and the moving air should have been used in finding Eq. 12*b*. The velocity of the air was not measured. Nevertheless, Eq. 12*b* seems to indicate that the floats were slowed down by air resistance.

A second set of broken lines (without points) is shown in Fig. 5. These lines were computed from Rehbock's revision of the Ganguillet-Kutter formula using $n = 0.010$. The flatter slope of these lines, compared to the slope of the full lines, indicates that there is an increased energy dissipation in the flow in steep chutes.

Supplementary Remarks on Mechanical Aeration

In the course of the experiments some interesting facts were revealed which bear on the causes of mechanical aeration. In Fig. 5 it is seen that the intersections of the lines for the Ganguillet-Kutter formula and the lines for the model tests occur at velocities from 10 to 11.5 ft per sec. A different choice of Kutter's n should alter the position of these intersections. However, it would seem that mechanical aeration begins at some definite velocity.

Again, in Fig. 4 it is seen that aeration is present down to a depth of 0.023 ft. This depth corresponds to a velocity of 12.5 ft per sec. When all such curves

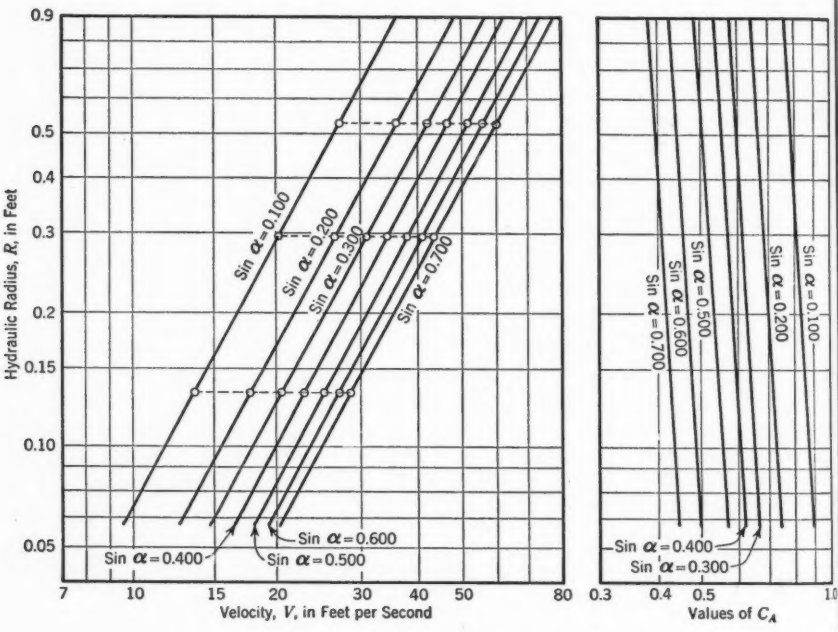

Fig. 6.—Logarithmic Representation of Two Formulas $V = f_1(R, S)$ and $C_A = f_2(R, S)$

were investigated, it was found that the velocity at which the proportion of air in the flow becomes unity varied between 11.5 and 14.8 ft per sec.

The following experiments were conducted to supplement these two observations:

(1) The end sections of the side-walls of a chute were removed and replaced by glass plates which extended beyond the end of the chute. The underside of the issuing jet thus could be seen clearly. The bottom velocity was increased until aeration of the under surface of the jet was visible. This bottom velocity was about 10 ft per sec.

(2) A similar experiment, in which a rectangular pipe was used (with glass plates replacing the two side-walls as in the preceding experiment), gave 10 ft

per sec as the velocity at which small droplets of water first began to separate at the exposed top and bottom surfaces of the jet. Below this velocity the jet remained intact.

It appears, therefore, that mechanical aeration begins at a velocity of about 10 ft per sec.

Conclusions

This investigation was undertaken to clarify the principles of flow in steep chutes. The average proportion of water in the flow at a cross section where the velocity is uniform is given by Eq. 2. Eqs. 10 and 11, shown graphically in Fig. 6, give, respectively, the mean velocity, V, and the proportion of water, C_A, in terms of the gross hydraulic radius, R, and the sine of the angle of inclination. These formulas are valid for chutes of dressed wood of approximately rectangular cross section.

DISTORTED RIVER MODELS [1]

BY A. ROHRINGER,[2] ESQ.

TRANSLATED AND ABSTRACTED BY DOUGLAS C. DAVIS,[3] JUN. AM. SOC. C. E.

A description of the methods used in the Hydraulic Laboratory of the Budapest (Hungary) Technical University in dealing with distorted river models may throw some light on the difficult problem of distorted river models.

The use of an undistorted scale in a large river model causes the phenomenon of laminar flow to take place, thus making the model dissimilar to the prototype structure. Assuming a 1 : 500 undistorted scale for a model

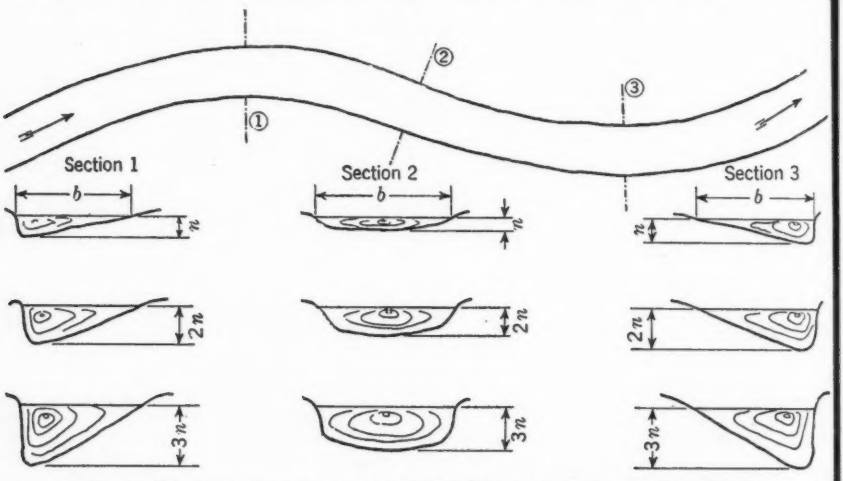

FIG. 1.—EFFECT OF DOUBLE AND TRIPLE DISTORTION ON TYPICAL CROSS SECTIONS

of the Danube River, the model depths of 1 to 2 cm would nullify any results obtained. Hence, the depth scale of such models is usually distorted. It is inadvisable, however, to increase the depth scale to more than three times that of the length and width since distortion exceeding this amount changes the flow conditions too radically. Fig. 1 illustrates the effect of double and triple distortion on typical cross sections. Although the depth scale may be distorted, the length and width scales should be equal to insure adequate similarity between prototype and model.

The proper simulation of bed-load material (geschiebe) in a river model is a difficult problem. A scalar reduction of the river geschiebe, in the model,

[1] Original source not known.
[2] Author's present position not known.
[3] Capt., Corps of Engrs., U. S. Army.

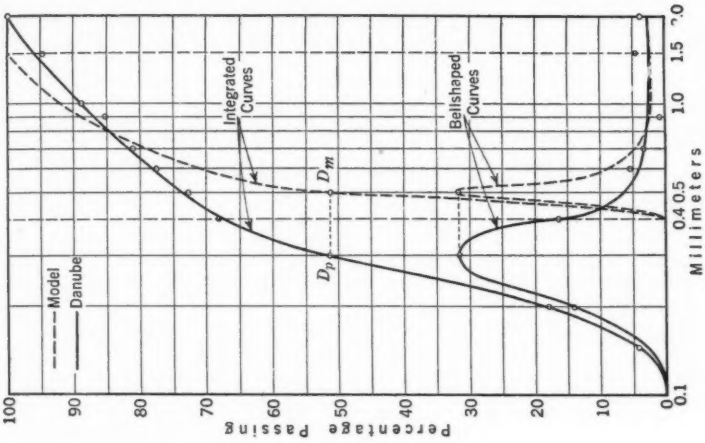

FIG. 3

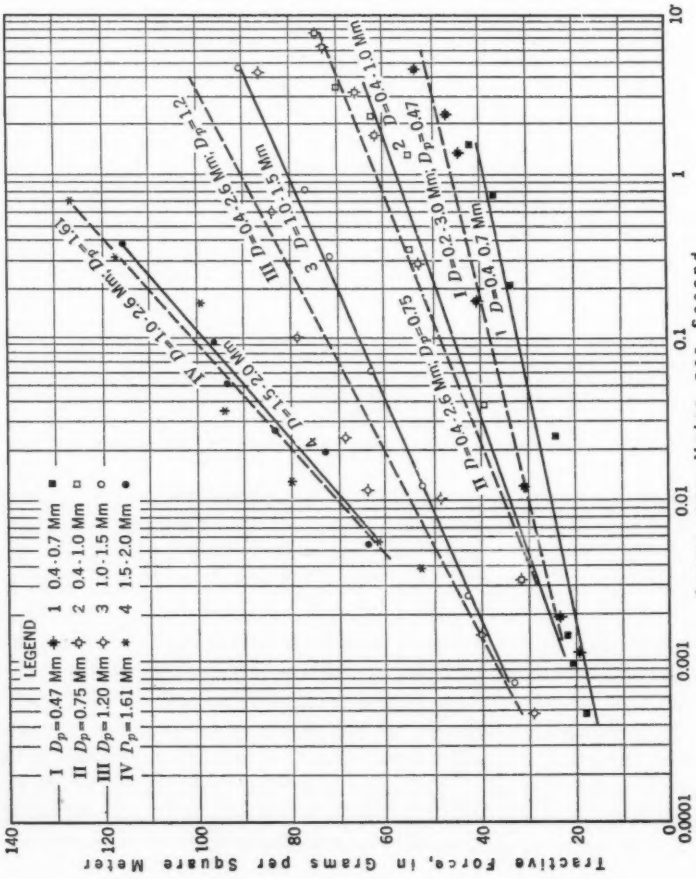
LEGEND
I D_p=0.47 Mm 1 0.4-0.7 Mm
II D_p=0.75 Mm 2 0.4-1.0 Mm
III D_p=1.20 Mm 3 1.0-1.5 Mm
IV D_p=1.61 Mm 4 1.5-2.0 Mm
I D=0.4-0.7 Mm
II D=0.4-1.0 Mm; D_p=0.75
III D=0.4-2.6 Mm; D_p=1.2
IV D=1.0-2.6 Mm; D_p=1.61
Tractive Force, in Grams per Square Meter
Grams per Meter per Second

FIG. 2.—GESCHIEBE LOAD (GRAMS PER METER PER SECOND)

results in a fine dust, the particles of which adhere to each other, forming a series of waves on the model bed—the so-called "ripple" formation. In the laboratory tests under discussion, the scale used for geschiebe was many times greater than the other scales. Of course, in such a case, the model slope must be increased to provide a velocity sufficient for proper geschiebe movement.

Since both geometric similarity and mechanical similarity have been abandoned through the use of depth distortion, there is no particular point in reducing the grain size of the geschiebe geometrically.

The securing of good crossings (of channel at bends) is an objective of prime importance in river models. In the absence of geometric and mechanical

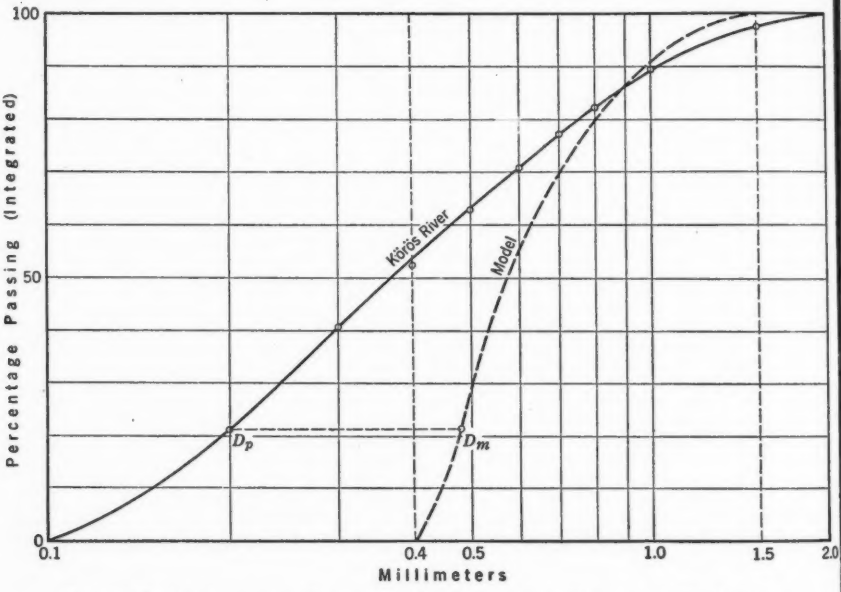

FIG. 4.—SIEVE ANALYSIS OF RIVER AND MODEL GESCHIEBE MATERIALS—KÖRÖS RIVER

similarity, Fargue's statement of the relationship between radius of curvature and depth provides an essential tie-in between model results and prototype behavior. According to this statement, curves of gradually decreasing radii develop gradually increasing depths. Thus, proper curvature, in both prototype and model, results in adequate depths and satisfactory crossings, regardless of the geschiebe material involved. With larger grain sizes, however, existing depths decrease, whereas, with smaller grain sizes, these depths increase. The selection of proper curves assures a thalweg suitable for ship traffic. A qualitative result is thus obtained which, assuming a good form (of channel), answers the purpose of a quantitative result. A little dredging in the prototype should complete the job. Thus, one concludes that a model study following Fargue's law assures a desirable result in the prototype regardless of the model geschiebe material used (within reasonable limits).

In the installation of training works, due consideration must be given to Girard's fundamental hypothesis—guide works on a concave bank should be reduced in height as they proceed toward the point of inflection. A height at the inflection point equal to that at the concave bend would influence the crossing adversely.

Definite criteria establish upper and lower limits for the model slope—the velocity (dependent on the slope) should be great enough to assure proper geschiebe movement, and yet should not approach such magnitude as to cause shooting flow. Either mixed or uniform-sized geschiebe material may be used in the model. Uniform material is preferable, in that the resistance to bed-load movement consists of masses of equal magnitude. The computation methods used by the late Allen Hazen, M. Am. Soc. C. E., and others[4] are suitable for the determination of a uniform grain size of model geschiebe material. The writer also has evolved a method[5] of determining proper grain size. The semilogarithmic plot shown in Fig. 2 enables one to make the proper selection.

Sieve analyses of river (Danube at Budapest) and model geschiebe material are given in Fig. 3. Eliminating the finest and coarsest grain sizes of the river material, the remainder is taken as the model material. The bell-shaped curves indicate the percentage weight distribution of grain sizes. The other curves are obtained by integrating the data of the bell-shaped curves. In

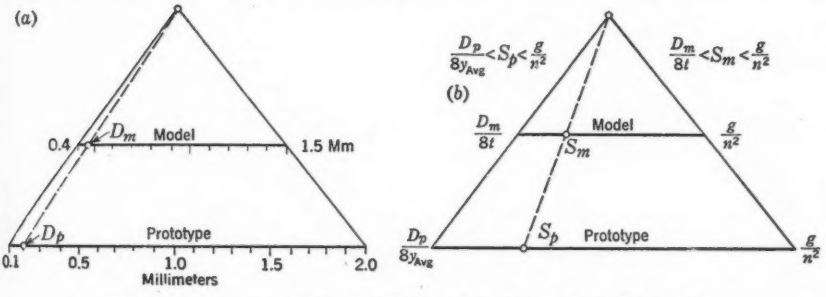

FIG. 5.—RELATION OF EFFECTIVE SIZE OF SEDIMENT—MODEL-PROTOTYPE

each case the inflection point of the integrated curve corresponds to the maximum of the bell-shaped curve. On the integrated curve for river material, this point determines the "standard" river grain size, D_p. Projecting this point on to the integrated model material curve gives the "standard" model grain size, D_m. This D_m is the grain size used by H. Krey and R. Winkel in their tests to determine slope boundaries. Fig. 4 shows similar distribution curves for river and model material of the Körös River in Hungary. The equation resulting from these tests follows:

$$\frac{D_p}{8\,y_{aver}} < S_p < \frac{g}{n^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

[4] "Die Grundbegriffe der Bodenmechanik und deren technische Anwendung mit besonderer Rücksicht auf den Wasserbau," by Janicsek Jaky, *Vizügyi közlemenyek*, 1933.

[5] *Proceedings*, International Ship Navigation Cong., Vol. XVI.

in which D_p = standard grain size of river material, y_{aver} = average depth, S_p = slope, g = acceleration of gravity, and n = Kutter's roughness coefficient. Eq. 1 defines the limits between which the slope S_p occurs. By applying these limiting values to the model conditions and using a proportional graphical construction as shown in Fig. 5, the model slope S_m is obtained.

The foregoing principles were applied in several model studies of the Danube. Since the river geschiebe material in the reach under investigation (below Budapest) was very fine (1 to 2 mm), a larger material was used in the model to avoid the troublesome "ripple" formation. A good navigation channel was obtained by adhering to the Fargue theory. The indications of the model study of the reach of the river above Baja, Hungary, in the so-called Bogyiszloer Cutoff—namely, that the extensive regulatory works originally proposed could be reduced considerably—were duly considered in improving the prototype. Model predictions were verified in the field and a substantial saving in the construction of regulatory works was effected. In this case, geometric similarity and mechanical similarity were replaced by Fargue's theory which, in so far as the main problem of the study (geschiebe movement) was concerned, proved entirely satisfactory.

Although an undistorted scale which is sufficiently large to insure turbulent flow is highly desirable, the cost is usually prohibitive; and, since distortion has proved practicable, there is no reason why it should not be used.

DEC 2 1 1943

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DECEMBER
1943

VOLUME 69 NO. 10

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AMERICAN SOCIETY OF CIVIL ENGINEERS

COMING MEETINGS

BOARD OF DIRECTION MEETING

January 17–18, 1944:

A Quarterly Meeting will be held in New York, N. Y.

April 17–18, 1944:

A Quarterly Meeting of the Board of Directors will be held at the Coronado Hotel, St. Louis, Mo. This is in conjunction with the Spring Meeting of the St. Louis Section, April 19–20, at the same place.

ANNUAL MEETING

NEW YORK, N. Y.

January 19–21, 1944

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 noon. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.